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OPTIMAT BLADES

TG 5

RESIDUAL STATIC STRENGTH OF FIBROUS COMPOSITES AFTER FATIGUE: A LITERATURE SURVEY

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1. EXECUTIVE SUMMARY

In the frame of Task Group 5 "Development of methodologies for residual strength and life prediction and condition assessment" of OPTIMAT BLADES project, University of Patras is the responsible partner for the preparation of a review reporting on residual strength assessment concepts for composites after fatigue, according to the work content of Task 13.1.

The present document is the first version of deliverable #6, "*Review of existing residual strength predictive models*" as described in ANNEX I of OPTIMAT BLADES contract. It is merely an initial literature survey and not a "review" since no critical appraisal of different existing theories is included. Several reasons have led to this outcome as for example:

(i) An unexpectedly large number of publications from different authors introducing "their" phenomenological residual strength theory.

(ii) Existing theories are in fact statistical models based on a number of hypotheses that are assumed valid upon corroboration of model predictions by experimental results. However, test data sets used for one model are not most of the times adequate for the implementation of another one and a thorough literature search for a unique test data set, to be used both for implementation and validation of a prescreened set of residual strength theoretical models was in vain, up to now.

(iii) Implementation of most theoretical models requires the development of software tools featuring statistical and multi-parametric optimization algorithms.

Therefore, the critical survey and validation of existing residual strength theories will follow, in the course of the OPTIMAT BLADES project, in future versions of this report and will be based, at least, on test results from the experiments described in the DPA of TG5.

2. INTRODUCTION

Wind turbine rotor blades are designed so as to withstand a high number of load cycles, order E+09, during their operational, 20 years, life. According to current design regulations, a safe life design methodology is followed and no account is taken of material property degradation and damage accumulation, which is known to be the case for the kind of composite materials used in blade manufacturing. The reason for not following a more advanced damage tolerant design approach can be attributed to lack of knowledge.

Several implications arise due to the above restriction; obviously blade designs are either conservative or cannot reliably fulfil the promise of 20 years operation under variable amplitude load cycles. On the other hand, since no account is taken of property degradation due to damage accumulation, it is not possible to assess structural blade response at a certain fraction of operational life, nor health-monitoring techniques can be developed that could lead to definition of periodic inspections. More important, however, is the fact that state-of-the-art life prediction codes are based on the linear Palmgren-Miner damage accumulation rule which for the case of spectral loading is known to yield results that in many occasions were shown to either under- or overestimate the expected life of the composite laminate.

In addition, dimensioning of wind turbine rotor blades is also based on stresses developed in the material during short time, extreme operating conditions. Design allowables used in the calculations are derived by dividing the static strength of the material with appropriate partial factors, for which it is not clear how they take into account material property degradation. Nevertheless, the question to be asked is if a rotor blade initially designed and certified for specific extreme load cases, continues to satisfy initial design requirements, after a fraction of its operational life, during which it is subjected to continuous variable amplitude cyclic loading, has been spent.

Clearly, a reliable engineering model capable of predicting degradation of static strength in composite laminates is required under the circumstances.
Consider the general case of a composite material specimen, which has for example an initial static tensile strength X on one axis and is subjected to a dynamic load history on the same axis. After a period of cyclic loading, one will observe that if the specimen is statically

tested, its strength X_r will decrease eventually. In all cases of structural materials, fatigue damage is correlated with this degradation of the actual static material strength. X_r is called residual static strength after fatigue or simply residual strength.

Residual strength, for a specific material, is in general a complicated function of the loading history. Assuming constant amplitude (CA) cyclic loading, residual strength is probably a function of loading frequency v, maximum cyclic stress, σ_{max}, stress ratio R, number of applied cycles n, and of course of the initial static strength X:

$$X_r = f(\sigma_{max}, R, n, v, X). \quad (1)$$

For the case of an anisotropic material system, e.g. a multidirectional laminate made of FRP, strength is expressed rather by a tensor than a scalar quantity, i.e. X_i instead of X.

Generalizing eq.(1) for the case of static strength degradation on one axis due to fatigue on another axis, or even due to a full plane stress, i.e. $\{\sigma_x, \sigma_y, \sigma_s\}$, loading history yields:

$$X_{r_i} = f_i(\sigma_{x_{max}}, \sigma_{y_{max}}, \sigma_{s_{max}}, R_x, R_y, R_s, n, X_1, X_2, \ldots), \quad (2)$$

where X_{r_i} denotes the i-th component of the residual strength tensor, expressed as a function, f_i, of in-plane fatigue parameters. Index i takes values 1, 2,..up to 5 or 9, etc. depending on material strength symmetry class. In this contracted tensor notation, X_1 stands for example for the tensile strength along axis-1 of the material symmetry system, X_2 for the tensile strength in direction-2, X_3 for the compressive strength in direction-1 etc.

Therefore, the engineering model required for residual strength of an anisotropic laminate is of the general form of eq.(2), assuming of course plane stress conditions and proportional loading, i.e. same number of cycles and at the same frequency for stress components σ_x, σ_y, σ_s. The problem is to define the form of functions f_i, based on a properly designed experimental program.

In the following sections, attempts to approach this target, already published in the open literature are presented. As shown, there are almost only 1D models, i.e. consideration of residual strength degradation in a material axis due to fatigue loading along this same direction.

3. RESIDUAL STRENGTH MODELS

3.1 State-of-the-art

A large number of theoretical models have been proposed to predict the residual strength of composite materials subjected to cyclic loading. The practical use of such models has been mainly fatigue life prediction, damage accumulation, proof testing etc. In these cases residual strength has been a simple means to macroscopically characterize material property degradation during operational life.

Residual strength studies were up to now mainly axial, i.e. the fact of the coupling between fatigue loading in one direction and residual strength degradation in another was ignored. As a consequence, investigations on residual strength degradation have been performed only for axial fatigue experiments. Recently, in a number of publications by Lessard et al., e.g. [1],[2], complex stress states are considered by testing off-axis specimens in order to evaluate a multi-axial life prediction methodology for UD laminates. With the exception of this research group, numerous investigators have proposed many different approaches on the subject of axial strength degradation under a variety of loading conditions and materials used.

These models can be broadly characterized as either mechanistic or phenomenological. Mechanistic models are those that quantitatively account for the progression of damage in composite materials, by taking into account the micromechanics of various damage modes induced by cyclic loading. J.C. Halpin [3] was the first to attempt (dominant crack propagation) implementation of such a model. Reifsnider introduced a philosophy on critical and sub-critical elements in the laminate [4],[5],[6]-[9], further investigated by researchers like Song and Otani [10]. Andersons and Korsgaard [11] investigated Reifsneider' s model, noticing that it cannot account for the apparent rate decrease of strength reduction at high life fractions, caused by failure of weaker specimen during cycling. Song and Otani [10] also tried to correlate critical strengths of the composite with various microdamage mechanisms, while Charewicz and Daniel approached residual strength using an unidentified damage function [12].
The above mechanistic models claim to offer a long-term promise; to be applicable to a wide variety of materials, layups and loadings with a minimal amount of experimentally obtained input. At present however, they are either in their infancy or have been applied to simple

fatigue loading or else include micromechanical parameters witch are difficult to obtain in structural engineering reality.

Phenomenological models on the other hand, are those that characterize residual strength in terms of macroscopically determined mechanical properties such as strength or stiffness. These models probably offer the most promising near term approach towards predicting residual strength degradation. Their major drawback is their dependency on large amounts of experimental data for each material, layup and loading of interest.

3.2 Phenomenological models

3.2.1 BROUTMAN & SAHU

Broutman and Sahu [13] presented probably the first approach on static strength degradation of GFRP composites in 1969. Later, in 1972, in their effort to develop a modified Palmgren-Miner rule, they presented a simple model to predict residual strength, based on linear strength degradation [14], with experiments performed on glass/epoxy cross-ply laminate. Keeping the same symbols as before (N denoting the characteristic life of the specimen at strength level σ_{max}) their equation can be written as:

$$X_r = X - (X - \sigma_{max})\left(\frac{n}{N}\right) \tag{3}$$

As Broutman and Sahu point out, this concept of linear strength reduction does not agree very well with the experimental results, especially in low fatigue stress levels, but it was the best possible given the number of just few residual strength tests performed in their study.

3.2.2 HAHN & KIM

In their studies [15] on fatigue life and proof testing of composites, Hahn and Kim introduced the concept of rate of change of residual strength. They assumed the time rate of decrease of residual strength is inversely proportional to the residual strength to a certain power. They also introduced an important assumption on the relation between static strength and fatigue life. This assumption was, that the specimen with the higher initial static strength would have also a longer fatigue life. In their own words: «A specimen of a certain rank in the fatigue life distribution is assumed to be equivalent in strength to the specimens of the same rank in the static strength distribution". Chou and Croman named this later on "strength-life equal rank assumption" [16]. This assumption was proved valid through proof testing [15], [17], [18].

In order to derive their residual strength relationship Hahn & Kim use the following assumptions:

a. The relation they proposed for residual strength is the following rate type equation:

$$\frac{dX_r}{dt} = -AX_r^{-(c-1)} \tag{4}$$

b. The positive parameter A(σ) depends on the applied load σ(t) and the exponent c is a material constant independent of X_r and t. Characteristics of the above equation are shown in Fig. 1. Depending on whether c is greater or lower than unity, the above equation represents a slow strength degradation followed by a rapid one, or the opposite. Fatigue failure occurs when the residual strength reduces to the maximum applied stress.
c. The above-mentioned strength-life equal rank assumption.
d. Static strength and fatigue life distribution are both fit by Weibull distributions as follows:

$$P_X(x) = \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \tag{5}$$

$$P_N(n) = \exp\left[-\left(\frac{n}{\tilde{N}}\right)^{\alpha_f}\right] \tag{6}$$

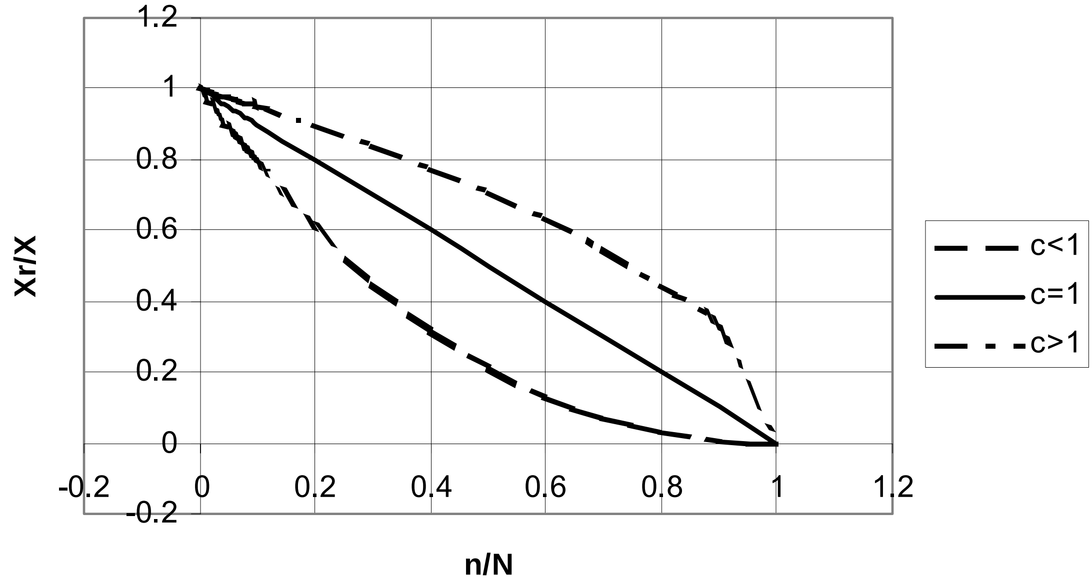

Fig. 1: Variation of residual strength degradation for different values of parameter c.

Integrating eq.(4) between the initial time t_o and t, considering the case of CA fatigue, yields:

$$X_r^{\ c} = X^c - cD(t - t_0) \tag{7}$$

Hahn & Kim point out that parameter D, which is the integral of A(σ) from t_o to t, in general depends on the characteristics of fatigue loading. In constant amplitude fatigue, appropriate parameters are the stress amplitude, stress ratio and frequency. In their case [15], all of the above are kept constant, so D is a constant also, while t and t_o can be replaced by load cycles n_o and n. Implementing assumption (c) into eq.(7) for t_o = n_o =0 one obtains the functional form of strength-life equal rank assumption:

$$N = \frac{\left(X^c - \sigma_{max}^c\right)}{cD} \tag{8}$$

Solving eq.(8) equation for cD and substituting in eq.(7), from n_o = 0 to n.

$$X_r^c = X^c - \left(X^c - \sigma_{max}^c\right)\frac{n}{N} \tag{9}$$

The probability distribution of residual strength is derived by substituting eq.(8) into eq.(5), considering that static strength distribution should be truncated by $\left(\frac{\sigma_{max}}{\beta}\right)^{\alpha}$,since only specimens with static strength greater then the maximum cyclic stress are finally tested for residual strength. The substitution leads to:

$$P_{X_r}(x_r) = \exp\left[-\left(\frac{x_r^{\,c} - cDn}{\beta^c}\right)^{\frac{a}{c}} + \left(\frac{\sigma_{max}}{\beta}\right)^{\alpha}\right] \tag{10}$$

To derive a fatigue life distribution Hahn & Kim use the following reasoning: Those specimens that survive n cycles will have strength greater than the applied stress σ_{max}. Therefore at the particular stress level, the probability of fatigue life being greater than N is equal to the probability of the residual strength being greater than σ_{max} when n=N:

$$P_N(n) = \exp\left[-\left(\frac{\sigma_{max}^c - cDn}{\beta^c}\right)^{\frac{a}{c}} + \left(\frac{\sigma_{max}}{\beta}\right)^{\alpha}\right] \tag{11}$$

Tests performed on Glass/Epoxy were used to evaluate model optimum parameter values for c and D, using least squares fit. Comparison of eq.(11) and the Weibull distribution fit of the fatigue life data indicated that eq.(11) is not good enough to predict the fatigue life distribution.

This model, combined with a stiffness degradation theory, was used lately by Whitworth to predict residual strength [19]. Experiments on $[\pm 35]_{2S}$ T300/5280 Graphite/Epoxy laminates proved it capable of predicting residual strength distribution.

Hahn and Kim' s model for c=1 leads to the linear degradation case of Broutman and Sahu.

3.2.3 YANG ET AL.

Yang and various associates are probably the research group most involved in studies on residual strength as a means for proof testing and life prediction of composite laminates for a variety of loading conditions. They have, since 1975, presented numerous publications based on the residual strength degradation model they developed [20]-[30]. From mid 80's, their publications were mainly oriented to stiffness degradation of composites due to fatigue [31], [32].

Their first attempt to model residual strength degradation was based on the assumption that residual strength is a monotonically decreasing function of load cycling, in a concept very similar to that proposed by Hahn and Kim [15]. Their reasoning is based on the following assumptions:

a. The rate equation describing residual strength is of the form:

$$\frac{dX_r(n)}{dn} = \frac{-f(\sigma_{max}, v, R)}{c X_r^{\,c-1}} \qquad (12)$$

in which $f(\sigma_{max}, v, R)$ is a function of the maximum cyclic stress, frequency and stress ratio, and c is a constant.

b. Static strength follows a 2-parameter Weibull distribution:

$$P_X(x) = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \qquad (13)$$

c. Fatigue failure occurs when the residual strength reduces to the maximum applied stress.

Integration of eq.(12) from 0 to n results to:

$$X_r^{\,c} = X^c - f(\sigma_{max}, v, R)n \qquad (14)$$

Using assumptions (b) and (c) along with the proposed degradation model, eq.(14), the following expression is obtained for the fatigue life N:

$$N = \frac{X^c - \sigma_{max}^c}{f(\sigma_{max}, v, R)} \tag{15}$$

Combining eqs (15) and (13) yields the probability distribution of fatigue life:

$$P_N(n) = 1 - \exp\left[-\left(\frac{n + \dfrac{\sigma_{max}^c}{f(\sigma_{max}, v, R)}}{\dfrac{\beta^c}{f(\sigma_{max}, v, R)}}\right)^{\frac{\alpha}{c}}\right] \tag{16}$$

Equating the scale parameter of this distribution with the characteristic life obtained from an S-N curve of the form:

$$KS^b\tilde{N} = 1 \tag{17}$$

One obtains $f(\sigma_{max}, v, R)$, which is [20]-[23],[25],[28]:

$$f(\sigma_{max}, v, R) = \beta^c KS^b \, . \tag{18}$$

In the above equation, S is the stress range, β is the scale parameter of the static strength distribution and K, b, and c are constants. The calculation of these last three parameters, once the static strength distribution of the material is known, is performed with a relatively limited experimental effort (30-40 fatigue and residual strength tests).

The final form of the residual strength degradation equation is:

$$X_r^{\,c}(n) = X^c - \beta^c KS^b n \tag{19}$$

The probability distribution of the residual strength after n cycles under a given stress level is derived using eqs (19) and (13).

$$P_{X_r}(x_r) = 1 - \exp\left[-\left(\frac{x_r{}^c + \beta^c K S^b n}{\beta^c} \right)^{\frac{\alpha}{c}} \right] \qquad (20)$$

This first model of Yang was validated experimentally for Graphite/Epoxy materials under various fatigue loading conditions: Tension- tension fatigue, R=0.1, for σ_{max} varying from 62% to 85% of the ultimate tensile stress [20], tension-compression fatigue, R from -0.15 to –0.3, and various σ_{max} [22], and shear loading, by testing axially $[\pm 45^o]_{2S}$ coupons [23], [25]. This model has been also applied in investigating load sequence effects using normal straight edge coupons [25] and bolted joints in composites [28]. Additionally, other researchers have also used this model of strength degradation [33].

In subsequent publications by Yang et al. [24], [27], the aforementioned model was generalized to include a wider class of composite materials, e.g. Glass/Epoxy, having a stronger degradation rate of the residual strength, than the previously studied Graphite/Epoxy.

The main new assumption on the theoretical derivation of the new model is the application of the "Strength-life equal rank assumption" that a specimen with higher ultimate static strength results in a higher fatigue life. This statistically implies that X and N are completely correlated so they are functionally related. This functional relation can be derived from the equation of the corresponding distribution functions of X and N.

Equating the two distributions, eqs (5) and (6), one obtains for the fatigue life:

$$N = \tilde{N} \left\{ \left(\frac{X}{\beta} \right)^{\frac{a}{a_f}} - \left(\frac{\sigma_{max}}{\beta} \right)^{\frac{a}{a_f}} \right\} \qquad (21)$$

Eq.(15) and eq.(21) yield:

$$f(\sigma_{max}, v, R) = \frac{1}{\tilde{N}} \cdot \frac{\left(\frac{X}{\beta}\right)^{c} - \left(\frac{\sigma_{max}}{\beta}\right)^{c}}{\left(\frac{X}{\beta}\right)^{\frac{a}{a_f}} - \left(\frac{\sigma_{max}}{\beta}\right)^{\frac{a}{a_f}}} \tag{22}$$

In this way, the resulting model includes also the statistical characteristics of the fatigue life. The final equation is:

$$X_r^{\ c}(n) = X^c - \frac{X^c - \sigma_{max}^c}{X^\omega - \sigma_{max}^\omega} \beta^\omega K S^b n \tag{23}$$

In the above equation $\omega = \frac{a}{a_f}$, while α and α_f are the shape parameter of static strength and fatigue life respectively.

This form is a more generalized residual strength degradation equation, since it includes the previous model of Yang for c=ω, the model of Hahn and Kim for ω=0, and the model of Broutman and Sahu for ω=0 and v=1.

The six model parameters are determined experimentally: α and β from static strength tests, b, ω and K from fatigue life data and c from residual strength tests. Experimental verification of this model was performed with test data from Glass/Epoxy coupons under dual stress level [27] and Graphite/Epoxy (proof loading studies) with satisfactory results. Andersons and Korsgaard also adopted this model in investigating high cycle fatigue of GRP composites, on a layup typical of wind turbine rotor blades [11].

The most recent and general form of Yang model can be found in [26], [29], [30]. One more parameter γ is introduced to the degradation rate equation of residual strength, reflecting the non linear dependence of the degradation rate on the number of fatigue cycles [26]:

$$\frac{d\left(\frac{X_r(n)}{\beta}\right)}{dn} = \frac{-\gamma n^{\gamma} f(\sigma_{max}, X)}{c\left(\frac{X_r}{\beta}\right)^{c-1}} \tag{24}$$

Integration leads to:

$$X_r^{\,c}(n) = X^c - \frac{X^c - \sigma_{max}^c}{(X^\omega - \sigma_{max}^\omega)^\gamma} \beta^{\omega\gamma} (KS^b n)^\gamma \tag{25}$$

In eq.(25), the expression of $f(\sigma_{max}, X)$ is derived by applying the fracture assumption to the integral of eq.(24), and replacing for the fatigue life the expression from eq.(21).

Model parameters are determined as above, while γ is defined, as ω, from residual strength test data.

3.2.4 CHOU & CROMAN

In the presentation of their theory [16], [34], Chou and Croman discuss on the restrictive assumptions of the models proposed by Hahn [15] and Yang (early model) [20]. Especially, they remark that this model contains only two parameters, $f(\sigma_{max}, v, R)$ and c, which are determined completely by the fatigue life distribution data. For a given material, under a given cyclic loading, once the static strength and fatigue life distributions are known, the residual strength at all fatigue cycles is known. In other words, according to these models, two materials having the same static strength and fatigue life distribution must also have the same residual strength distribution.

In order to overpass this restriction, they propose, on one hand a different wear-out model including an additional free parameter [16], and on the other hand they introduce the sudden-death model [34], as a limiting case in residual strength study.

Their degradation equation is based on the following assumptions:

a. Static strength and fatigue life are two-parameter Weibull distributed:

$$P_X(x) = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \tag{26}$$

$$P_N(n) = 1 - \exp\left[-\left(\frac{n}{\tilde{N}}\right)^{\alpha_f}\right] \tag{27}$$

b. The strength-life equal rank assumption, already introduced by Hahn and Kim, in the form of eq.(28). In this equation $P_{X,\sigma_{max}}(x_\gamma)$ is the conditional probability of all the specimen with initial static strength greater than the maximum applied stress.

$$P_{X,\sigma_{max}}(x_\gamma) = P_N(n_\gamma) \tag{28}$$

c. The previously used fracture condition: $X_r(0) = X_\gamma$ and $X_r(n_\gamma) = \sigma_{max}$

Assumptions (a) and (b) yield:

$$n_{\gamma}^{\alpha_f} = x_y^a - \sigma_{max}^a \qquad (29)$$

In the above equation, static strength and life are normalized by their Weibull scale parameters β and $\tilde{N}$ respectively, while α and α_f are their respective shape parameters. n_γ is the number of cycles at which the residual strength is measured, and at this life only 100γ percent of the specimens survive. Combining assumption (c) and eq.(29) yields:

$$X_r{}^a = X_\gamma^a - n_\gamma^{a_f}\left(\frac{n}{n_\gamma}\right)^{ia_f} \qquad (30)$$

X_γ is the minimum initial static strength required, so that the specimen survives at least n_γ cycles. In other words X_γ and n_γ are the static strength and fatigue life respectively, that give a cumulative distribution value of 1-γ. By assuming different values for i, a family of degradation curves is obtained, ranging from slow wear-out to sudden degradation, see Fig.2.

The case for which the residual static strength remains constant, i.e. independent of load cycles, until very near to N and then drops suddenly is expressed through the “sudden death model” [34].

The probability distribution of residual strength derived by Chou and Croman [16] can be computed only numerically.

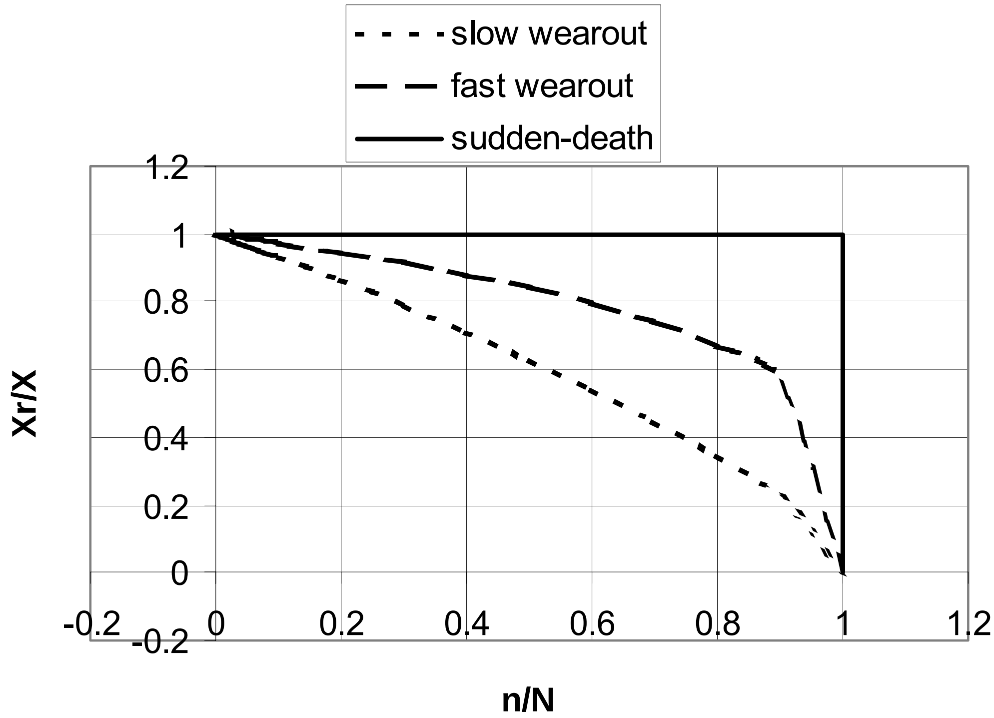

Fig. 2: Transition of residual strength models from slow wear out to sudden death.

3.2.5 HARRIS ET AL.

Bryan Harris research group have published a number of papers on the fatigue behaviour of various types of composite materials [35]-[37]. Their work includes life prediction of Carbon T800 [36] as well as GRP, CFRP and KFRP exposed to various environmental conditions [35]. They have also investigated the fatigue behaviour of hybrid Carbon/E Glass/Kevlar composites [37]. On reference [35] they query the validity of the wear out model of Broutman and Sahu, stating that it oversimplifies to some extend the sequence of events immediately proceeding failure. In their first approach, they use the resemblance of residual strength test on GRP with the decay curve in viscoelasticity or relaxation, to introduce the following rate equation for residual strength, which they call the "decay model":

$$\frac{dX_r}{dn} = -BX_r \qquad (31)$$

In the above equation B is a positive constant, depending mainly on σ_{max}. In particular, it was deduced, after a number of tests on various stress levels, that B is a linear function of σ_{max}. Integrating eq.(31) yields:

$$\log B = \log D + C\sigma_{max} \qquad (32)$$

$$X_r = X \exp\left\{D \exp\left(C\sigma_{max}\right)\right\} \qquad (33)$$

Their second approach, called "interaction model", is motivated by the apparent similarity of the residual strength curves at various stress levels, and the possibility that some appropriate normalizing factor may take account of the stress dependence on the rate of damage accumulation. On that purpose they introduce the residual strength ratio as:

$$r = \frac{X_r - \sigma_{max}}{X - \sigma_{max}} \qquad (34)$$

As well as the cycle (or log-time) ratio:

$$t = \frac{\log n - \log 0.5}{\log N - \log 0.5} \quad (35)$$

These two are combined with the condition that the boundary points are (1,0) and (0,1) in the following:

$$t^x + r^y = 1 \quad (36)$$

Where *x* and *y* are determined as the 'best fit' to the data, using various fitting methods they propose in [35].

3.2.6 LESSARD ET AL.

Lessard, Shokrieh and other researchers have been working on various aspects of the fatigue behaviour of composites, like damage and failure analysis [38],[39]. However, the most interesting part of their work, in connection with residual strength, is the combination of various theories in order to make a model capable of predicting fatigue life of unidirectional plies, under plane stress state, based on uniaxial fatigue data [1],[2],[40], called "Generalized residual material property degradation model".

In [1] there is a review of the existing residual strength degradation models. The one they adopted is in fact the model by Harris et al, with some insignificant modifications. The equation used is:

$$X_r(n,\sigma_{max},R) = \left[1-\left(\frac{\log(n)-\log(0.25)}{\log(N)-\log(0.25)}\right)^{\beta}\right]^{\frac{1}{\alpha}}\left(X-\sigma_{max}\right)+\sigma_{max} \qquad (37)$$

This residual static strength is used afterwards to replace the static strength term in the denominator of a polynomial failure criterion of the form:

$$\left(\frac{\sigma_{22}}{X_{r22}}\right)^2+\left(\frac{\sigma_{12}}{X_{r12}}\right)^2=1 \qquad (38)$$

The above equation includes only the residual strength in the transverse to the fiber direction and shear residual strength, which is the case of matrix-dominated failure.

Consequently, eq.(38) ends up in a relation predicting the number of cycles to failure, under any given plane stress fatigue loading conditions [1]. For the determination of model parameters, static and cyclic tests under both transverse and in-plane shear loading conditions have been performed (considering matrix dominated failure as mentioned above). The model was evaluated experimentally on off-axis specimens of AS4/3501-6 Graphite/Epoxy UD laminates and the results were promising.

3.2.7 SCHAFF AND DAVIDSON

Schaff and Davidson have adopted in their studies on life prediction [41], [42] the following assumptions:

a. The wearout equation of residual strength is of the following form.

$$X_r(n) = X - f(X, \sigma_{max}, R) n^v \qquad (39)$$

b. Both fatigue life and static strength are represented by two parameter Weibull distributions of the form of eq.(26),(27)

c. The previously mentioned failure condition.

In eq.(39) v is called 'strength degradation parameter'. By imposing the failure condition, assumption (c), the above equation takes its final form:

$$X_{r(n)} = X - (X - \sigma_{max}) \left(\frac{n}{N}\right)^v \qquad (40)$$

The above equation is identical to the one derived by Reifsnider [4],[5],[6]-[9]. The difference is that Reifsnider uses a micromechanical approach based on critical and subcritical element to derive the equation, while Schaff and Davidson use it without any physical interpretation in order to fit their experimental data in a pure phenomenological sense.

An innovative point in their paper is the formulation of the residual strength Weibull probability distribution with a shape parameter that is not constant but depends linearly on the number of load cycles. This is to take into account that residual strength distribution is getting more disperse at high cycle numbers. The equation of the shape parameter, β, is based on the observation that it is bounded by the Weibull shape parameters of static strength, β_s, and fatigue life β_t. The final form of the CDF is:

$$P\left[X_r \le x\right] = 1 - \exp\left[-\left(\frac{x}{X - \left(X - \sigma_{max}\right)\left(\frac{n}{N}\right)^{v}} \right)^{\beta_s - \left(\beta_s - \beta_t\right)\left(\frac{n}{N}\right)} \right] \tag{41}$$

The parameter v is determined by trying to match the above single parametric CDF with an experimental one.

Schaff and Davidson have used Broutman and Sahu's experimental data to validate their model, first in one stress level fatigue and then in two stress levels and spectrum loading. The prediction was deemed satisfactory in most cases, but parameter v seems to vary significantly with σ_{max}: From 0.75 for 241 MPa to 2.0 for 338 MPa.

3.2.8 SENDECKYJ

In a review chapter on life prediction of composite materials [43], Sendeckyj is referred extensively on residual strength theories as a means of predicting life under constant and variable amplitude fatigue, as well as their statistical interpretations. He resumes a number of the previously released residual strength theories and presents 6 models, based on the following assumptions:

a. The static strength is two parameter Weibull distributed
b. The residual static strength X_r after n cycles of constant amplitude loading is related to the initial static X, by a deterministic equation of the form:

$$\frac{dX_r}{dn} = -\left(\frac{1}{\gamma}\right) f \sigma_{max}^{\gamma} X_r^{(1-\gamma)} \tag{42}$$

Where f and γ are two dimensionless functions that do not depend on stress.

c. Fatigue failure occurs when the residual strength decreases to the maximum applied stress.

Integration of eq.(42), given the residual strength, yields the initial static strength for failure during the first cycle:

$$X = \sigma_{max}\left[\left(\frac{X_r}{\sigma_{max}}\right)^{\frac{1}{S}} + (n-1)f\right]^{S} \tag{43}$$

where γ has been replaced by 1/S. Eq.(43) leads to different residual strength degradation models for different expressions of the parameters S and f. Sendeckyj presents six such models, see Table1.

Table 1:Residual strength degradation theories. S_0, D, C, G are model parameters

	S	f
W1	S_0	1
W2	S_0	C
W3	S_0	$C(1-R)^G$
W3A	$S_0(1-R)^G$	$C(1-R)^G$
W4	$S_0+D(1-R)^G$	$C(1-R)^G$
W4A	$S_0(1-R)^D$	$C(1-R)^G$

According to Sendeckyj's own words:

"Fatigue model W1 is the classical power law fatigue criterion giving a straight line representation of the S-N curve on a log-log plot. It is the simplest model that one can use. Fatigue model W2 is the wearout model in the form used by Sendeckyj [44]. Since it assumes that S and f are constant, it can only be used to correlate fatigue data at a specified stress ratio. Fatigue model W3 is a three parameter fatigue model that is similar to those used by Yang and coworkers [20]-[30] and Whitney [45],[46]. It is the simplest fatigue model that can take into account the stress ratio dependence. It gives the same asymptotic slope for the S-N curve at different stress ratios, thus it does not model the dependence of the slope of the S-N curve on R. Models W4 and W4A are four parameters models that attempt to account for the R-dependence of the slope of the S-N curve. As it was shown, these fatigue models do not yield unique estimates of the model parameters. Finally, fatigue model W3A is a special case of fatigue model W4A. It is a three-parameter fatigue model that takes into account the R-dependence of the slope of the S-N curves. Moreover it corresponds to one of the optimum selections of parameters in model W4A."

Applying assumption (c) upon the general expression of the residual strength model, eq.(43), we obtain the shape of the S-N curve associated with the assumed residual strength degradation model:

$$X = \sigma_{max}\left[1 + (N-1)f\right]^{S} \tag{44}$$

The probability distribution of fatigue life is derived from assumption (a) and eq.(44), considering conditional probabilities to account for the fact that the initial static strength should be greater than the maximum applied stress:

$$P(N/X \geq \sigma_{max}) = \exp\left\{\left(\frac{\sigma_{max}}{\beta}\right)^{\alpha} - \left(\frac{N - \left(-\frac{1-f}{f}\right)}{\frac{\left(\frac{\beta}{\sigma_{max}}\right)^{\frac{1}{S}}}{f}}\right)^{S\alpha}\right\} \tag{45}$$

The probability distribution of residual strength is derived from assumption (a) and eq.(43), considering also conditional probabilities to account for the fact that the residual strength should be greater than the maximum applied stress:

$$P(X_r / X \geq \sigma_n) = \exp\left\{\left(\frac{\sigma_n}{\beta}\right)^{\alpha} - \left(\left(\frac{X^r}{\beta}\right)^{\frac{1}{S}} + \left(\frac{\sigma_{max}}{\beta}\right)^{\frac{1}{S}} f(n-1)\right)^{S\alpha}\right\} \tag{46}$$

where

$$\sigma_n = \sigma_{max}\left[1 + (n-1)f\right]^{S}. \tag{47}$$

4. CONCLUSIONS

A literature survey on residual strength models was presented in this report. Emphasis was given on pioneering works in the field, where theoretical models were first established and validated through adequate experimental evidence. There is a portion of published work, especially in conference proceedings where various researchers adopt some pre-existing model to fit their experimental data. No citation is given to such work, unless it is related to wind turbine rotor blade materials, e.g. [11].

As stated in the introduction, the report in its present form is not a review on the subject. It will become such in the future, as each model will be implemented in software and mutual comparisons, as well as with OPTIMAT experimental data, will highlight inherent advantages or deficiencies. After all, with phenomenological theories the only valid qualification criterion is the corroboration of theoretical predictions by experimental data. Such a critical appraisal of residual strength theories is not available at the moment, to the authors' knowledge.

It should be emphasized, once again, that all existing theories are of 1D formulation. UP in the frame of OPTIMAT TG5 is the responsible partner to formulate a valid 2D residual strength degradation theory, an ambitious task at least.

The time this report was edited, additional new theories were discovered in recent publications [47], [48]. In a later version of this report these theories will be included along eventually with others that will be suggested by our collaborators in TG5.

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