Benchmark of Lifetime Prediction Methodologies

OB_TG1_R012 rev. 001
doc. no. 10218

Draft version, September 2004
Confidential

OPTIMAT BLADES

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TG 1

Change record

Issue/revision	date	pages	Summary of changes
000	September 30th, 2004	33	na
001	November 30th, 2005		included comment on outcome of TG6 discussion on inclusion of UTS in S-N curves

Introduction

This document contains a description of the methodologies and the results of the benchmark calculations performed by DLR (German Aerospace Institute), UP (University of Patras), and WMC (knowledge center Wind turbine Materials and Constructions) in the frame of the Benchmark exercise in TG1 (WP 3.1).

The objective of this exercise is, to synchronize the lifetime prediction methodologies used by the different labs. A lifetime prediction calculation consists of several steps, and different methods can be used in each step, resulting in prediction differences that potentially are solely based on details in the calculation method (and not on fundamental differences). Once the laboratories are confident, that the methodologies they use produce the same results for the same input, influences of fundamentally different approaches can be more easily identified and more efficiently analysed. The benchmark exercise was extended slightly, to give a critical appraisal of the available data reduction methods and existing lifetime prediction methodologies. As such, it contributes to the development of design rules for variable amplitude loading, which is the focus of the remainder of the work in TG1.

The set-up of the report follows the chronology of the lifetime prediction methods it describes. It starts with briefly discussing the rainflow counting algorithm. Then, methods of deriving S-N curves, which describe the relationship between lifetime and applied stress is covered in some detail. Not only linear regression results are shown, also methods as have been devised by Whitney and Sendeckyj are implemented and the results are compared among the participating laboratories. Finally, some of the results discussed in these paragraphs are used in lifetime predictions for WISPER and WISPERX. In this last chapter, two different CLD (Constant Life Diagram) definitions are taken into account, and their influence on the lifetime predictions is evaluated.

Counting Method

There are different methods of isolating cycles from a variable amplitude spectrum, of which Rainflow counting is the most established one for time series of fatigue signals. A note on Rainflow counting is in order here: classically, two slightly, but fundamentally different algorithms have coexisted since the publication of Downing and Socie's 'simple rainflow counting algorithms' [1]. These algorithms give similar results as the algorithms invented initially, and simultaneously, by Endo&Matsuishi and de Jong [2, 3]. Only when the original signal is rearranged to start and end with the maximum peak or minimum valley ('cyclic' rainflow count), the results are identical. In this case, no half cycles will be counted. In a non-cyclic rainflow count (i.e., according to the original algorithm), half cycles are possible, and differences in the results may occur, which may have considerable influence, especially for the larger stress ranges.

The Rainflow counting algorithms of the three laboratories have been used to count the WISPER and the WISPERX standardised wind turbine load spectra. See Appendix IV for the results. For completeness, the counting results are given both in non-cyclic and cyclic form. The results are given in terms of cycle range, minimum, and number of half cycles. For the benchmark, the cyclic counting results were compared across laboratories, and found to be equal. In this benchmark, the cyclic Rainflow count method was used as an input to the lifetime predictions of a later chapter. For completeness, also two tables are included giving the 'from-to' matrix of the rainflow count result. The differences between a cyclic and a non-cyclic count of the WISPER(X) spectra are indicated with red shading in the relevant matrix positions.

These tables serve as an input to the lifetime prediction described later on in this document, along with the results of the S-N curve definition, which are discussed next.

S-N curve definition

Two fixed datasets were analysed to obtain relevant S-N curves. R=0.1 data from ECN (taken from the FACT database), and preliminary tests from the OPTIMAT blades project. The full datasets are shown in Appendix I.

Three methods of S-N curve definition were used, viz. linear regression on a lin-log, and a log-log scale; Whitney's method (essentially lin-log regression using Weibull statistics instead of Normal statistics); Sendeckyj's method (using Weibull statistics and a wear-out model).

Below, the methods are briefly described. Especially the Sendeckyj and the Whitney methods are not straightforward analytical methods, so the corresponding parameters of the S-N-curves had to be generated numerically. The resulting S-N-curves are compared, and the influence of incorporating particular datapoints, such as static data, or run-outs is described. A detailed description of the computational background is given for some of the methods in Appendix III.

Linear regression

Linear regression is a fairly standard method, the results were generated using MS EXCEL's 'Linest' or 'Trendline' function or using a dedicated FORTRAN code. Linear regression was performed on the OPTIMAT sample data, excluding static data, and using a log-log type S-N diagram. This is of the form:

$$\log(\sigma_{\max}) = d\log(N) + c \text{, or } \sigma_{\max} = KN^{-\frac{1}{b}} \qquad (1a, 1b)$$

In all cases, N was the dependent variable, although it is usually plotted on the abscissa. Number of cycles to failure as a dependent variable is believed to be physically more correct than the lay-out of a classical S-N diagram suggests.
The results from Table I suggest, that the linear regression is done in a (very) similar way by the different laboratories, yielding identical results.

Table I: Comparison of linear regression parameters, delivered by participating laboratories, using log-log type expression (see eq.) and excluding static data, N dependent variable

	DLR			**UP**			**WMC**		
R	-1	0.1	10	-1	0.1	10	-1	0.1	10
B	9.39	9.12	29.07	9.39	9.10	29.08	9.39	9.10	29.08
1/b	0.107	0.110	0.034	0.107	0.110	0.034	0.107	0.110	0.034
K	574.00	846.99	428.47	574.40	847.25	428.89	574.49	847.11	428.54

Sometimes, a lin-log, or exponential S-N curve is used to describe S-N-data. This is of the form:

$$\sigma_{\max} = a\log(N) + b \qquad (2)$$

The data were also evaluated using this expression, see Table II and figures 1-4.

Include static data or not?

In the above, the regression was done excluding static data. The question if static data should be included or excluded from the regression is not trivial. First of all, the question is, if a static test can be considered as a special case of a fatigue test.

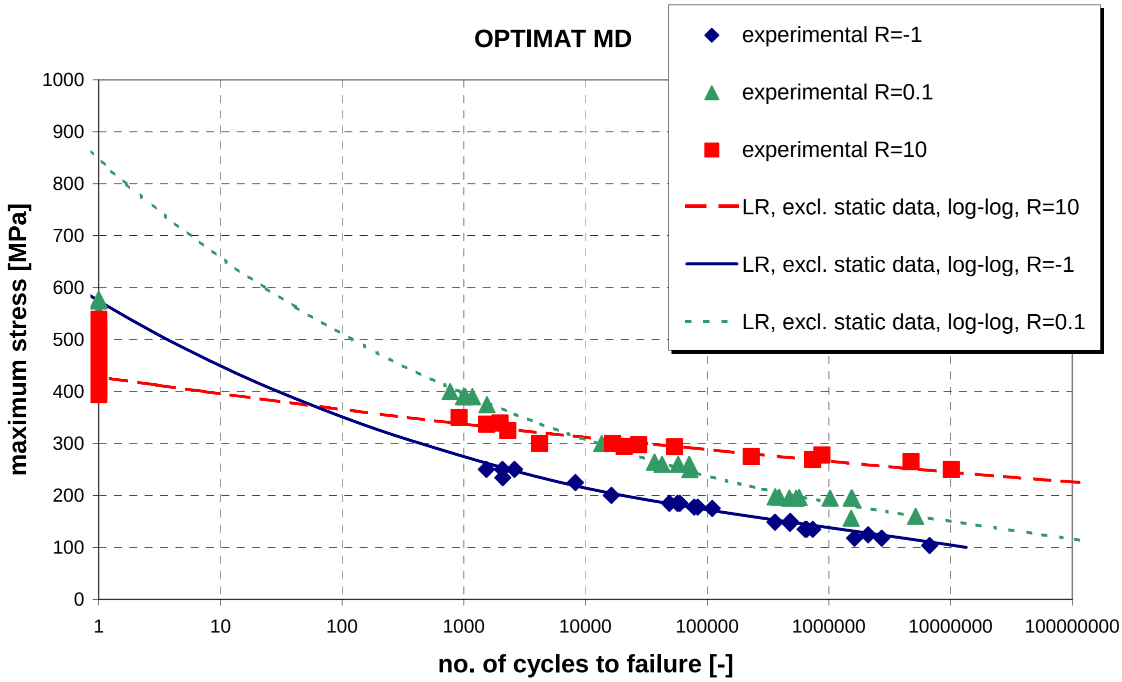

Figure 1: Linear regression (LR), excluding static data, using log-log S-N curve

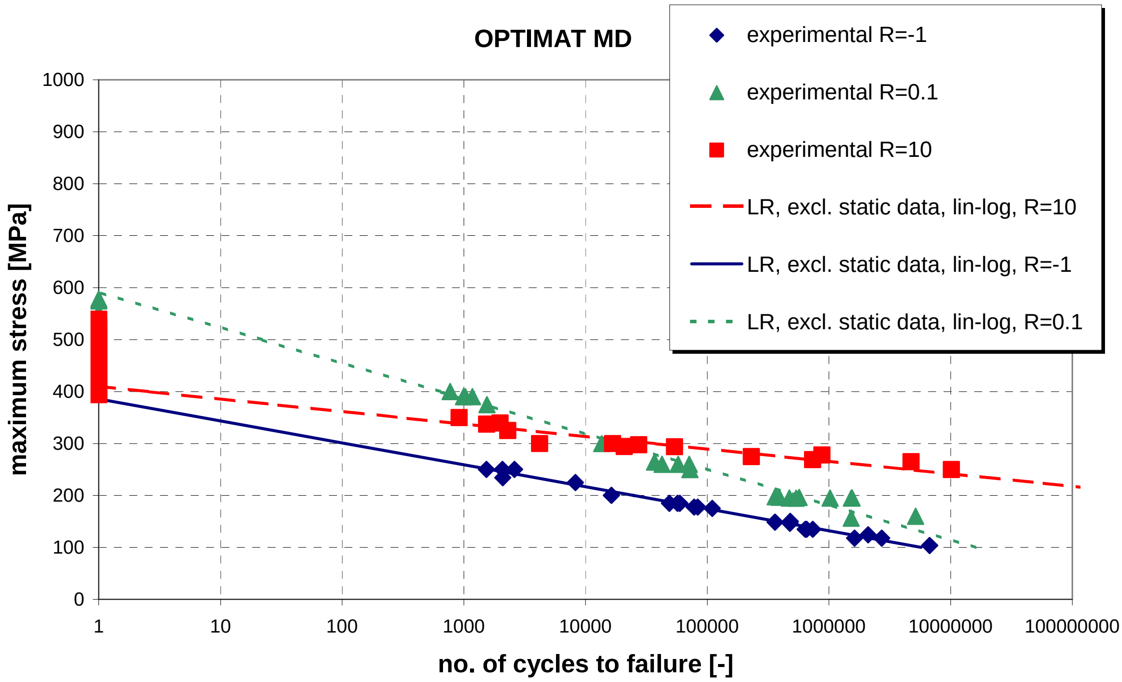

Figure 2: Linear regression (LR), excluding static data, using lin-log S-N curve

First, let us consider the assumption that static data should not play a part in determining the S-N curve. Strictly speaking, the validity of a regression cannot be guaranteed outside the region where

Table II: Regression parameters

	excl. static data				incl. Static data			
	log-log		lin-log		log-log		lin-log	
R	**d**	**c**	**a**	**b**	**d**	**c**	**a**	**b**
10	-0.0344	2.6320	-24.0054	409.5882	-	-	-	-
-1	-0.1065	2.7593	-42.2840	385.7580	-	-	-	-
0.1	-0.1099	2.9279	-68.2087	591.1971	-0.0737	2.7331	-57.2649	531.6509
0.1	-	-	-	-	-0.0986	2.8688	-88.6240	694.9499*
	(Figure 1)		(Figure 2)		(Figure 3, 5)		(Figure 4, 6)	
*This row shows values using high-strain rate static tests [4]								

it is applied. Since the data collected are in the range 100<N<1000000, this means, that regression may not be appropriate for describing very low-cycle fatigue, as is seen in Figs 1 and 2. Although for R=-1 and R=10, the static data roughly coincide with the y-intercept of the regression line, for R=0.1 the extrapolation does not seem appropriate. For a lin-log regression line, neither of the static data seem to be correlated well with the y-intercept of the regression line. Note, that if the regression included static data, Ultimate Tensile strength (UTS) was used for R=0.1, and Ultimate Compressive Strength (UCS) for R=-1 and R=10.
It should be noted, that in these plots, the static data were obtained at displacement rates which were roughly 100 to a 1000 times lower than the maximum displacement rates in fatigue loading. A short pilot study has been carried out [4], where tensile tests were performed at strain rates, comparable to those seen in fatigue. These static data are shown in Figs. 3 and 4, for regression excluding static data, with R=0.1 only. There is a clear strain rate influence, and the maximum stress is higher than the y-intercept of the R=0.1 regression line. Despite the fact that the test conditions were very similar in terms of strain rate and geometry, the ‘prediction’ of static strengths by the extrapolated regression line deviates equally far from the static data as in the case of ‘low-strain rate’ static data.
When a regression line is used in a CLD to do lifetime predictions, these predictions could be considered less valid due to the poor agreement with experimental data in the low-cycle régime. In the particular case of R=0.1, the prediction based on this R-ratio can be expected to be slightly conservative (with respect to the high-strain rate data).

Under the assumption that static data *should* be a part of the S-N determination process, the problem of inaccurate prediction could be solved by including static data in the regression, see Figs. 5 and 6. For a log-log regression, the high-strain rate static data seem to facilitate a reasonable regression line, although lifetime predictions may be slightly non-conservative in the high-cycle régime due to a smaller slope of the regression line relative to the data. A lin-log regression forces the regression line to a larger slope than the fatigue data show.

From the limited data available, there is no clear indication that static data should be plotted in an S-N diagram or included in a regression. Including static data in a regression might force the slope to be incorrect, which affects the entire range of predicted lifetimes. Excluding static data seems to help the correct description of fatigue data, but in lifetime predictions might lead to inaccuracies in the low-cycle régime, which especially affects load spectra with a few high-load peaks (such as WISPER(X)).

The reason why static tests at strain rates comparable to fatigue tests seem inappropriate in an S-N diagram was not pursued, but it must be emphasized that only very limited tests were done (no compressive tests, single strain rate datum, ca. 10 specimens. Moreover, the differences in failure mode between static and fatigue were not thoroughly investigated.

Note, that in the final stage of the OPTIMAT programme, it was recommended that static data should only be included in an S-N diagram for informative purposes [13].

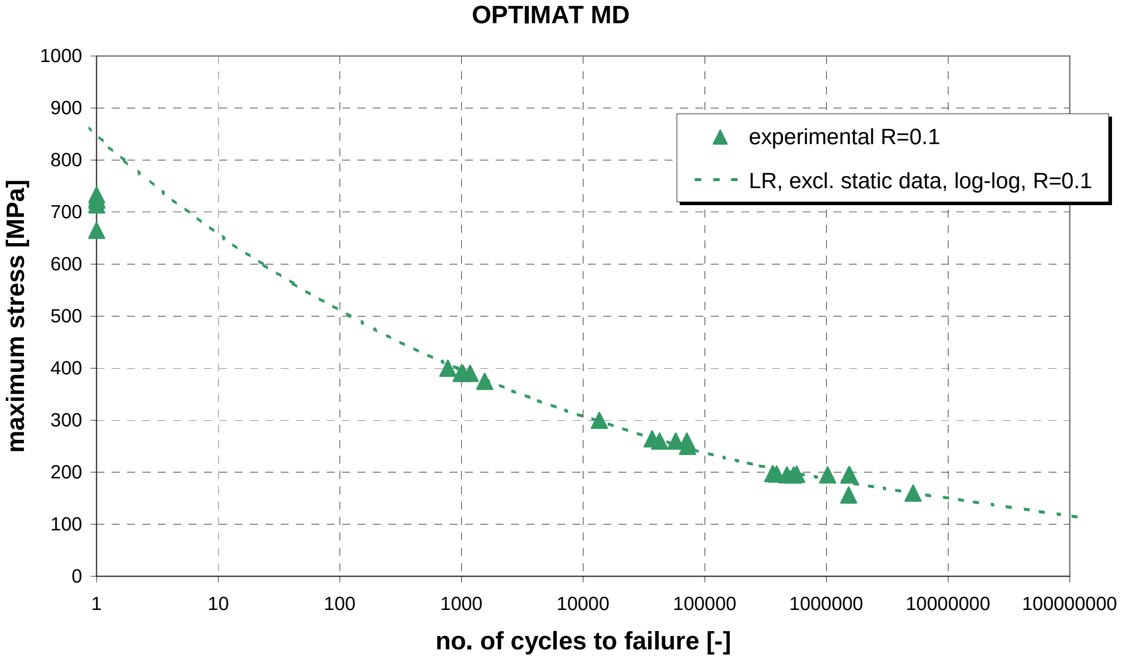

Figure 3: Linear regression (LR, log-log), excluding static data, ‘fast’ static results shown

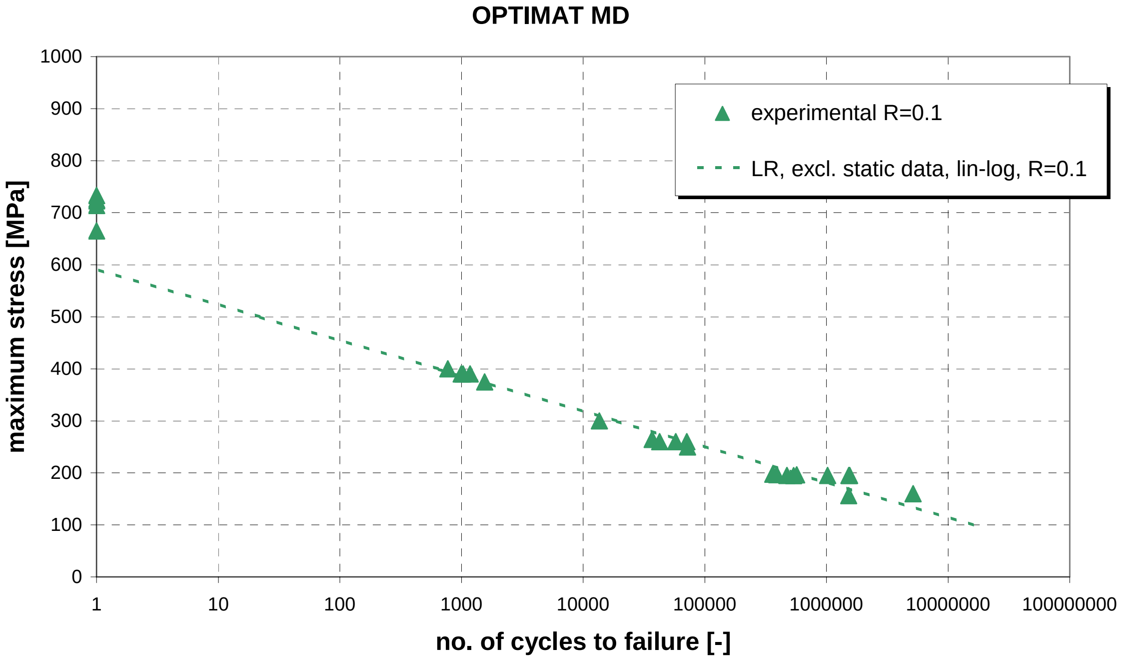

Figure 4: Linear regression (LR, lin-log), excluding static data, ‘fast’ static results shown

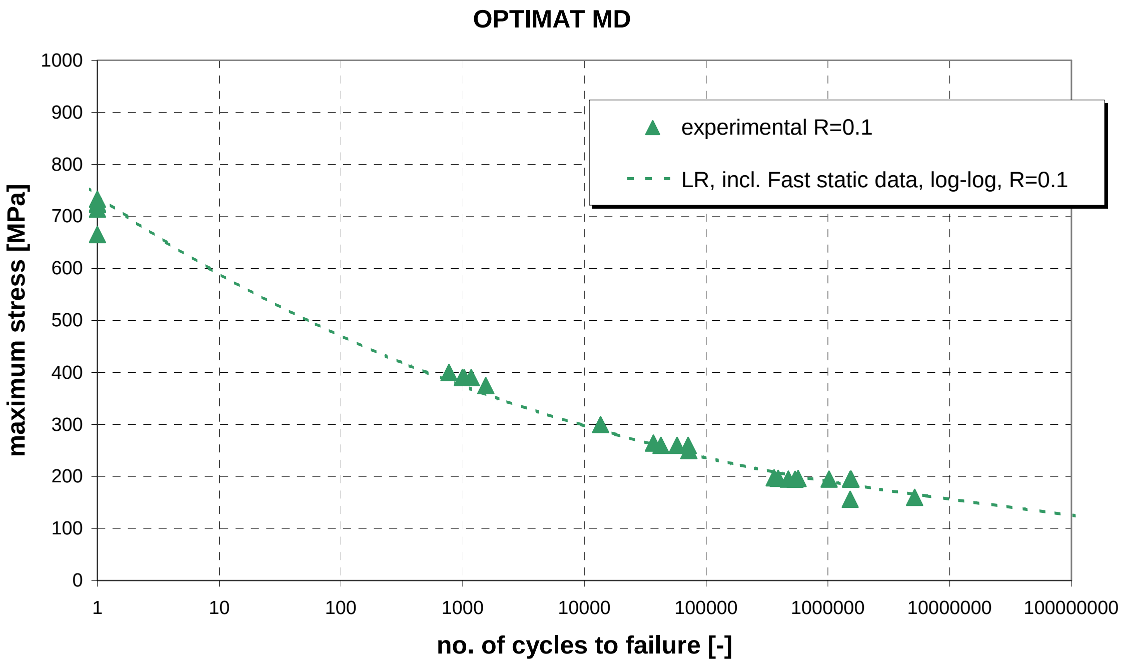

Figure 5: Linear regression (LR, log-log), including ‘fast’ static data

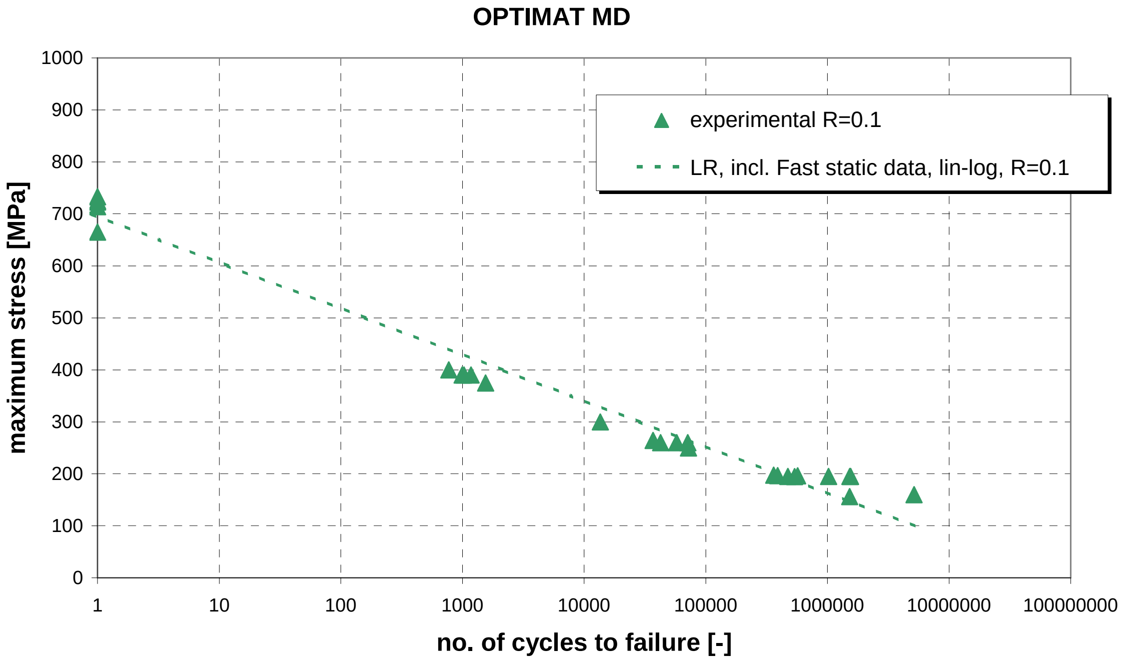

Figure 6: Linear regression (LR, lin-log), including ‘fast’ static data

Other S-N curve formulations
The question posed in the previous paragraph can be answered (with ‘yes’) by adopting an S-N curve formulation that allows for an S-shaped S-N curve, which could accurately describe both the static data and the constant amplitude data.
One candidate for such an S-N formulation found in the literature is derived from the ‘wearout model’ by Sendeckyj [5](which will also be discussed later in this document):

$$\sigma_e = \sigma_a (1 - C + CN)^S \tag{3}$$

Where σ_e is equivalent static strength, σ_a maximum applied stress, N no. of cycles to failure and C and S are fitting parameters. Finding the mean S-N curve, i.e. for probability of failure = 50%, means that σ_e can be set to the ultimate strength US, and the equation can be rewritten to:

$$\sigma_a = \frac{US}{(1 + [N - 1]C)^S} \tag{4}$$

Another option is the three-parameter formulation by Epaarachchi [6], that was recently adopted by Sutherland and Mandell [7]:

$$US - \sigma_a = a\sigma_a \left(\frac{\sigma_a}{US}\right)^b (N^c - 1) \tag{5}$$

with a, b, and c fitting parameters. Rewriting this to make N explicit:

$$N = \left(\frac{US - \sigma_a}{a\sigma_a \left(\frac{\sigma_a}{US}\right)^b} + 1 \right)^{\frac{1}{c}} \tag{6}$$

Sutherland and Mandell have formulated fitting parameters for 13 R-ratios for their ‘DD16’-material, but they do not describe the fitting procedure in detail.

Tolerance bounds
Whitney and Sendeckyj used Weibull statistics to derive reliability curves instead of mean S-N curves from their data. A similar result can be obtained for the methods described above using ‘Normal statistics’. The regression line is used to create a dataset of residuals from the experimental data, for which the standard deviation is determined. This can be used to estimate e.g. the lower 95/95 tolerance bound. For details see [8].

Whitney

Whitney has published a paper where he fits a log-log (power law) S-N curve to a set of fatigue data, similar to the above. However, he does not use ‘Normal statistics’ linear regression, but Weibull statistics. His procedure for data reduction,as described in [9], consists of three distinct phases:

1. Finding the Weibull shape parameter, α, and the scale parameter N, per stress level
2. Pooling the data, assuming that the Weibull parameters are independent of stress level
3. Finding the Weibull parameters for the complete fatigue dataset

Using the Whitney method was facilitated by the development of a software routine including an iteration loop to find the most appropriate shape parameter per level, and after pooling, for the pooled data. This routine was then used on available data.
By definition, Whitney does not include static data. Also, at least two datapoints per stress level are needed for the algorithm to work. Preferably, more data points are needed, the pooling scheme helps to 'enlarge' the database from which the Weibull parameters are found.
Catering for run-outs, Whitney includes a correction on the pooling scheme for 'type I censoring'. This means that the tests are terminated at a predefined number of cyles. As a consequence, if one stress level contains only two points, of which one is a run-out, and the user does not choose to use run-outs, this level is discarded altogether in the process of finding α. The reason is, that it is not possible to derive an α or N_{0i} for a single point, hence data cannot be normalized at this level and used to find the overall α and N_0.
Note, that in the currently used algorithm, the pooling scheme as suggested by Whitney was slightly generalized to accommodate for different values for the run-out number of cycles (whereas Whitney assumed that at each stress level, the predetermined number of cycles at which the test was terminated was always the same). Thus, equation 23 and 24 from [9] were rewritten to:

$$\frac{\sum_{j=1}^{n_i} N_{ij}^{\hat{\alpha}_{f_i}} \ln N_{ij}}{\sum_{j=1}^{n_i} N_{ij}^{\hat{\alpha}_{f_i}}} - \frac{1}{r_i}\sum_{j=1}^{r_i} \ln N_{ij} - \frac{1}{\hat{\alpha}_{f_i}} = 0 \qquad (7a)$$

$$N_{0_i} = \left\{ \frac{1}{n_i} \sum_{j=1}^{n_i} N_{ij}^{\hat{\alpha}_{f_i}} \right\}^{\frac{1}{\hat{\alpha}_{f_i}}} \qquad (7b)$$

(Note, that the only difference is the r_i-term in 7a). Two datasets have been distributed during the course of the benchmark; first, the OPTIMAT MD constant amplitude data from the basic S-N curve programme (see Appendix I, and figures1-6), and data from a previous project, taken from the wind turbine materials database 'FACT' [10]. The latter data were distinctly divided in levels, whereas the OPTIMAT MD data slightly deviate from their targeted levels due to small variations in specimen thickness. For use of the Whitney method on these data, the data were slightly adapted (i.e. data close to each other in terms of stress level were stratified to the average stress for these data). This was only successful for R=0.1 data; the other S-N data were too fragmented. See Appendix II for the original data (and the slightly transformed OPTIMAT MD data). The results are presented in Table III. From these results, it seems that the algorithms implemented by the different laboratories yield very similar results. It was found that some variation of the results occurred as a function of precision chosen in determining α .

Since Whitney uses no static data and a log-log S-N curve, it is to be expected that the results from this procedure, for a reliability level of 0.5, are close to the linear regression in a log-log S-N

Table III: Results for Whitney's method

OPTIMAT MD				**ECN dataset**								
	R=0.1			**R=0.1 (excl. 110 MPa)**			**R=0.1 (incl. 110 MPa)**			**R=-1**		
	DLR	**UP**	**WMC**	**DLR**	**UP**	**WMC**	**DLR**	**UP**	**WMC**	**DLR**	**UP**	**WMC**
α	2.4078	2.4086	2.41			3.5		3.4	3.58		2.0157	2.02
B	9.2732	9.306	9.4671	11.09		11.113		10.68	10.619		10.37	10.342
1/b	0.1078	0.1074	0.1056	0.0901		0.0899		0.0936	0.0942		0.0964	0.0967
K	841.43	837.69	825.18	579.37		574.94		597.22	596.49		431.18	431.49
R(N)	0.95	0.95	0.95			0.95		0.95	0.95		0.95	0.95
Prec.			10^{-6}			10^{-6}			10^{-6}			10^{-6}

diagram excluding static data. Figure 7 shows, that this is indeed the case (the curves are virtually on top of each other).
If this benchmark were considered as a ranking exercise for choosing the 'best method' to fit data, the considerable additional computational effort needed for the MLE does not seem to be justified from this last example. Worse, the Whitney-curve deviates slightly from the (physically more correct) regression with N as a dependent variable. On the other hand, Whitney is more suitable when Weibull statistics are preferred.

The method used in his original paper could be extended to other S-N curve definitions, such as a lin-log line.

Sendeckyj

Sendeckyj's method is fundamentally different from linear regression and Whitney's method in the sense that it makes use of the SLERA (Strength-Life-Equal-Rank-Assumption), thus assuming that fatigue life is uniquely related to initial static strength and residual strength. Sendeckyj's method consequently also allows using residual strength data for determining constant amplitude behaviour.
As an example, Sendeckyj included a dataset with some residual strength datapoints in his paper. As a test, this dataset has been investigated using WMC's algorithm (programmed according to the paper), and as can be seen from Appendix III, the results correspond considerably well to the values in the paper.
The results for some parameters of the sample data that were considered in the benchmark seem to be sensitive to the algorithm used. This is especially true for the C-parameter, and for the scale parameter. Also, this seems to be especially true for datasets with no static data. For the results, look at Table IV and figures 8-12. These figures show the mean S-N curves for the parameters in table IV. The solid lines represent curves including static data, the dashed curves represent those without static data.

Partly, the (mostly small) difference in results can be attributed to the numerical precision with which the algorithm searches for a maximum alpha. In some datasets, the relationship between the shape parameter and the C-parameter is very flat, i.e. there is no distinct global maximum and there is a large range of values for C for which the shape parameter is close to the maximum value, as was also noticed by OK [11]. Thus, maximum shape factor is not sensitive to the C-parameter. The S-N curve can be very sensitive to the parameter C, however. As an example, look at the data for OPTIMAT MD, R=0.1, excl. static data. WMC obtains a maximum of 22.983 vs 22.984 for DLR. The shape factors are practically identical, but the scale factors are almost a factor of two apart, leading to a rather different S-N curve. A similar case is the ECN 0.1 data without static data, for which DLR has calculated 2 sets of results. Although the shape factor is only 0.02% different, the S-N curves again look very different, see figure 11.

Although in this section the wearout model from Sendeckyj's original paper was used, the method can be readily extended to other two-parameter S-N definitions, such as the lin-log or log-log S-N curves.

Table IV: Results of Sendeckyj parameter fit on sample data sets

	OPTIMAT MD data														
	R=0.1						**R=10**			**R=-1**					
	DLR		**UP***		**WMC**		**DLR**	**UP***	**WMC**	**DLR**		**UP***		**WMC**	
	incl. ST	**excl. ST**	**incl. ST**	**excl. ST**	**incl. ST**	**excl. ST**	**excl. ST**	**excl. ST**	**excl. ST**	**incl. ST**	**excl. ST**	**incl. ST**	**excl. ST**	**incl. ST**	**excl. ST**
α	24.931	22.984	29.148		24.932	22.983	40.086	39.990	40.087	17.757	46.250	15.262		17.626	46.242
Beta	537.30	489.40	539.19		537.32	793.22	426.12	418.99	427.38	520.64	265.35	514.32		520.13	264.96
C	0.0278	0.0100	0.0194		0.0281	1.4801	1.0000	0.7283	1.0955	0.5502	0.0004	0.4171		0.2157	0.0004
S	0.0988	0.0999	0.1038		0.0987	0.0987	0.0325	0.0322	0.0325	0.1033	0.1156	0.1051		0.1124	0.1160
	ECN data														
α		26.314	22.034	26.314	22.034	26.314					21.222		21.219		21.219
Beta		573.44	415.41	573.20	415.41	586.51					290.62		290.72		290.57
C		0.8950	0.0250	0.8940	0.0254	1.1522					0.0092		0.0093		0.0091
S		0.0906	0.0911	0.0906	0.0912	0.0906					0.1038		0.1037		0.1039
	alpha=99.98% alpha_max														
α		26.310													
Beta		518.19													
C		0.3													
S		0.0906													

***Results based on MATLAB optimization schemes (different scheme was used for ECN data)**

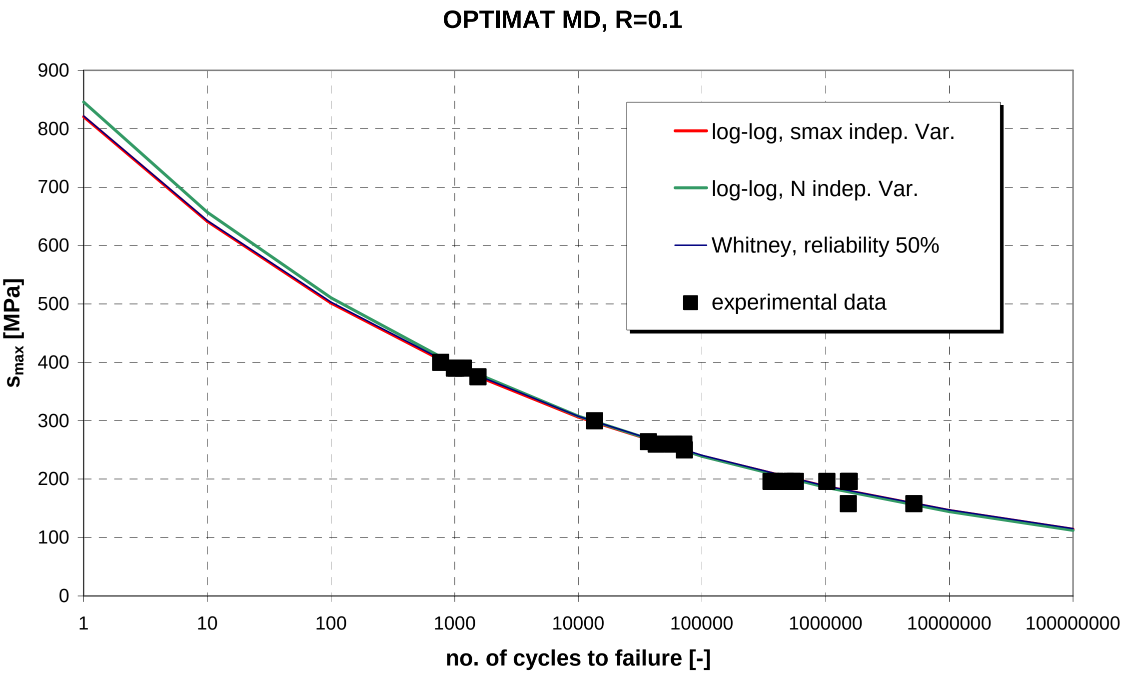

Figure 7: Whitney results compared to linear regression S-N curves

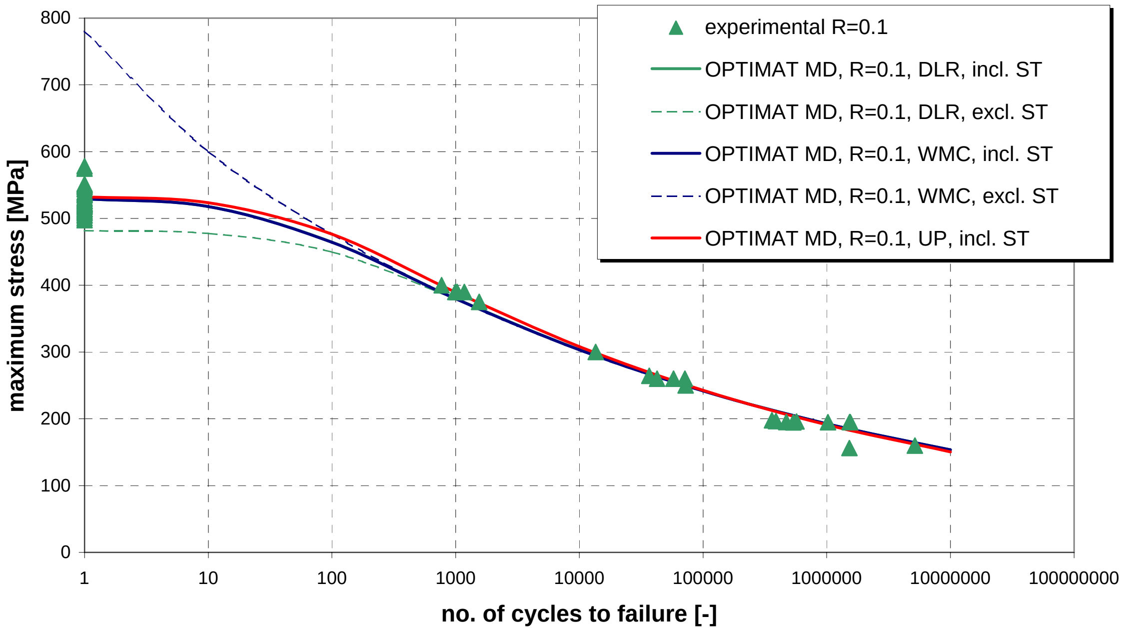

Figure 8: Sendeckyj S-N curves (1)

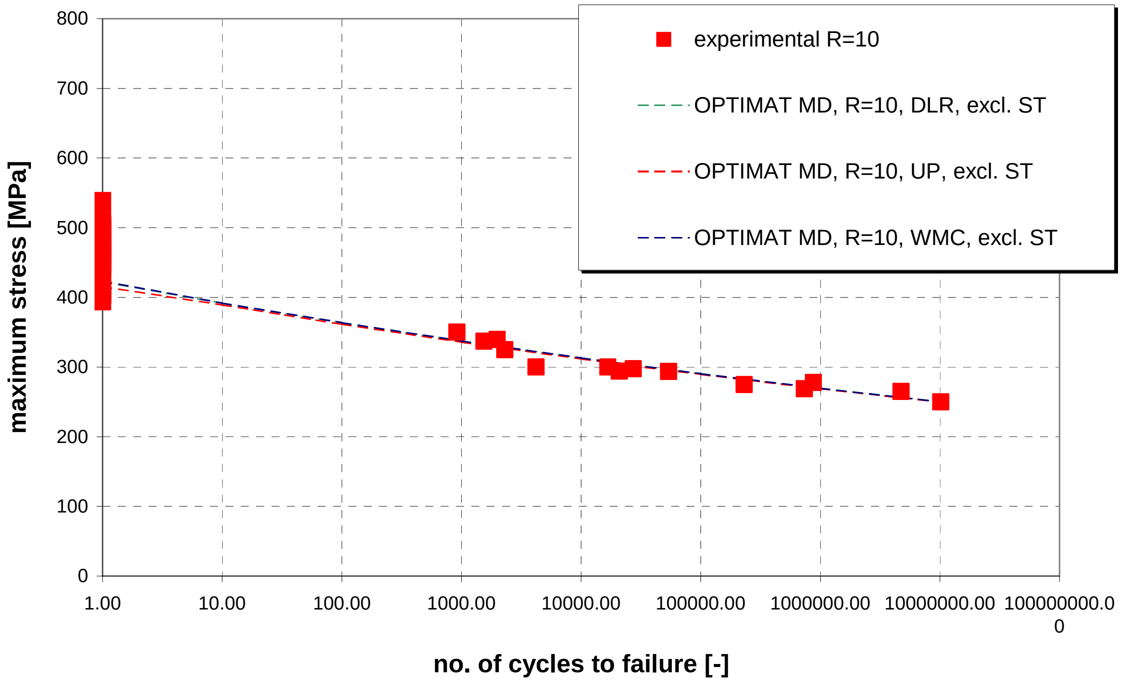

Figure 9: Sendeckyj S-N curves (2)

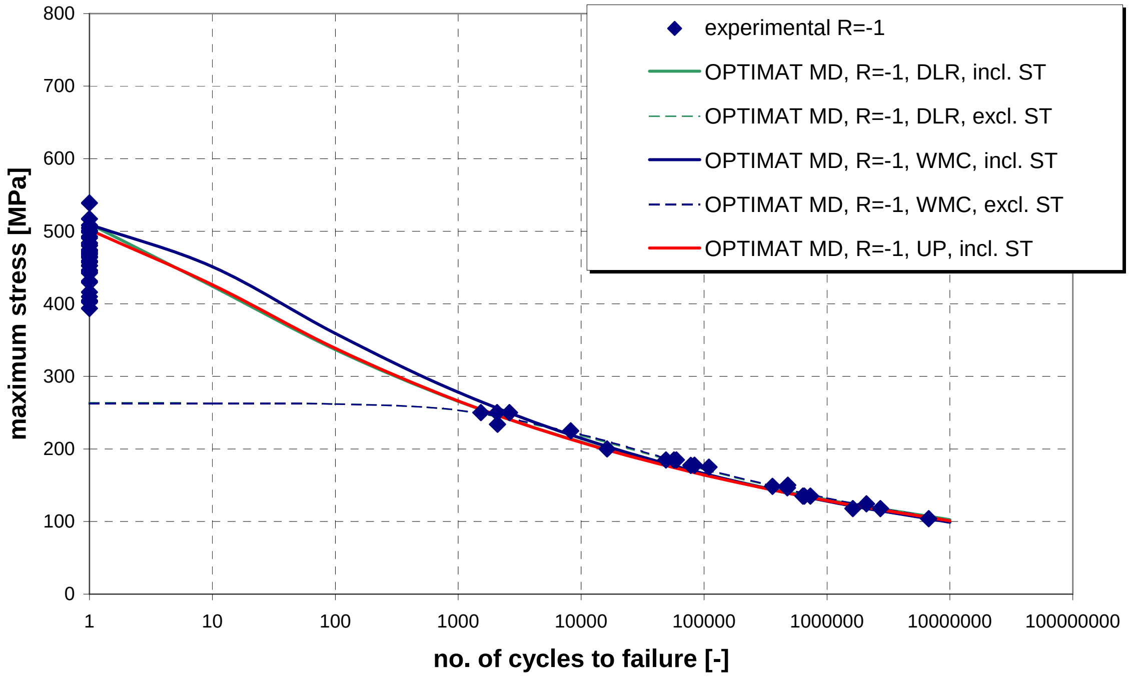

Figure 10: Sendeckyj S-N curves (3)

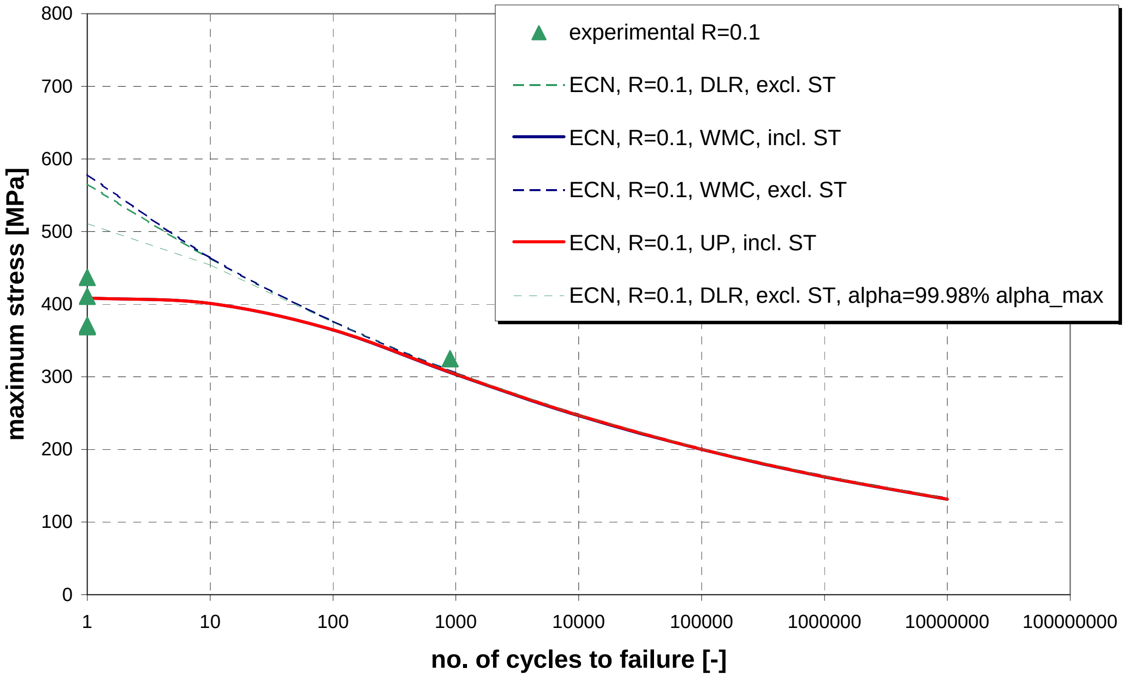

Figure 11: Sendeckyj S-N curves (4)

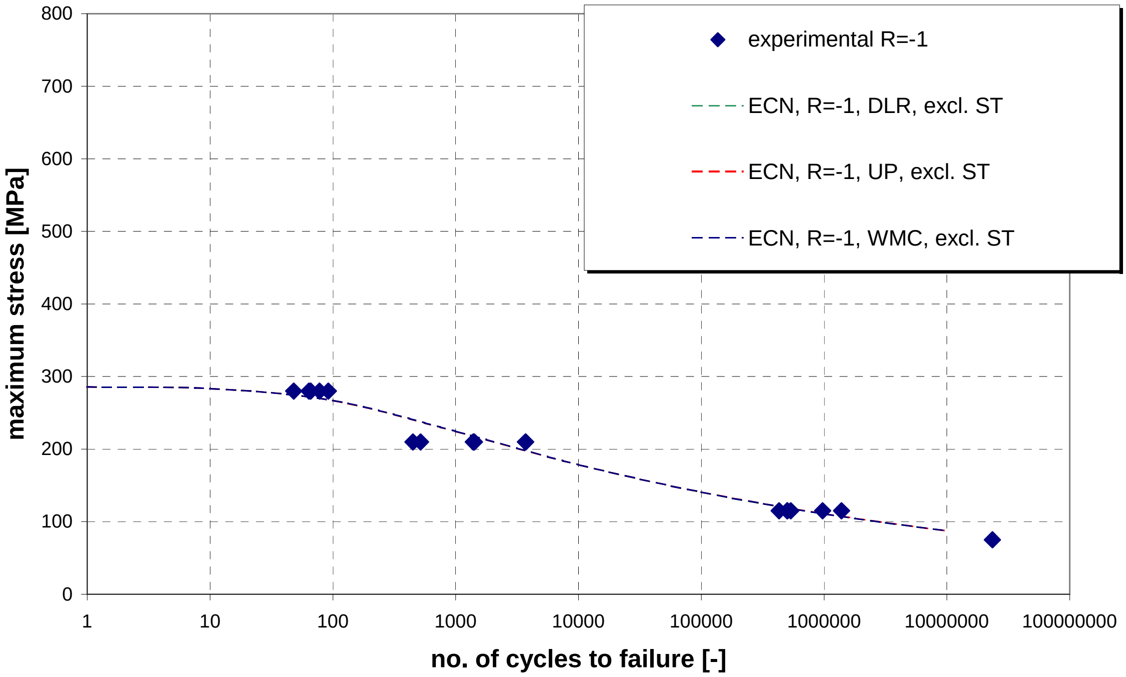

Figure 12: Sendeckyj S-N curves (5)

Lifetime predictions

The lifetime predictions discussed in this chapter, use some of the above S-N curves, Rainflow counting, and Linear Miner's summation:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{8}$$

where
D = Damage parameter (failure occurs when D=1)
k = number of cycle types (combination of mean stress and amplitude)
n = number of cycles of type I in the counting results
N = number of allowable cycles (taken from the appropriate CLD)

As the above has clarified, there are different ways to (rainflow-)count load spectra, and to define S-N curves from given datasets. The S-N curves are converted to Constant Life Diagrams, from which any combination of mean stress and stress amplitude can be related to an allowable number of cycles.
Three different shapes of a Constant Life Diagram (CLD) are used: the Linear Goodman diagram, the 'Shifted' linear Goodman diagram, and the bi-linear Goodman diagram.

The classical Linear Goodman diagram is the most commonly used CLD, because of its simplicity. For any cycle type with mean stress s_{mean} and stress amplitude s_{amp}, an equivalent stress amplitude s_{eq} at R=-1 is derived according to:

$$s_{eq} = \frac{s_{amp} \cdot UTS}{UTS - s_{mean}} \text{, for } s_{mean}>0\text{, and} \tag{9a}$$

$$s_{eq} = \frac{s_{amp} \cdot UCS}{UCS - s_{mean}} \text{, for } s_{mean}<0 \tag{9b}$$

These formulas follow from figure 13. In this figure, the abscissa represents both s_{mean} and R=1. The ordinate represents the R=-1 line. The other lines are lines that connect points in the s_{mean},

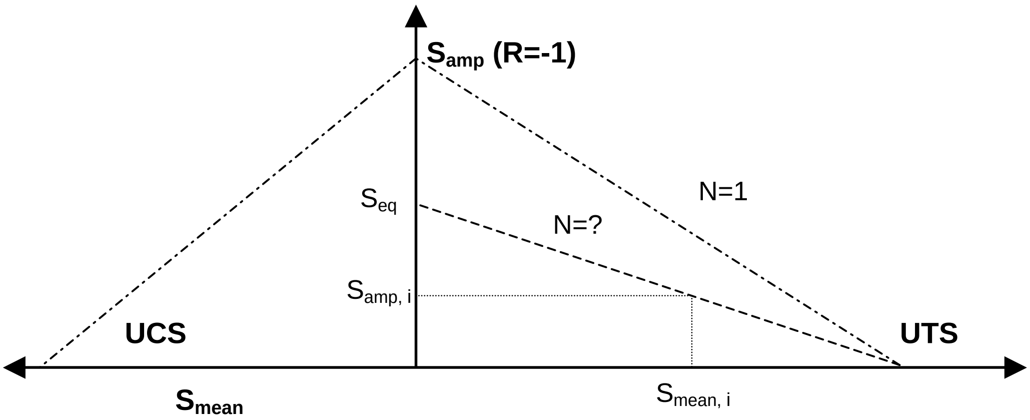

Figure 13 : Linear Goodman Diagram

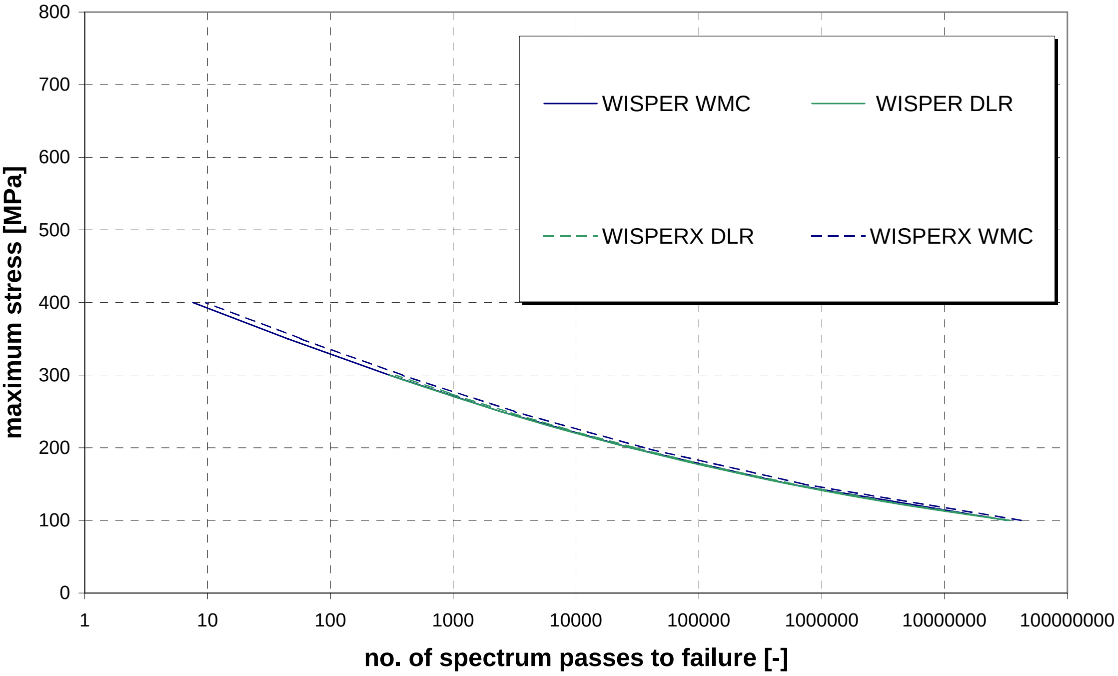

Figure 14: Lifetime predictions using Linear Goodman Diagram

s_{amp}-space with equal lifetime. As a bounding constant life line, the N=1 line is also drawn. The equivalent stress, found from these formulations, is then input in any S-N definition describing the S-N curve at R=-1, and the allowable number of cycles N_i for this particular cycle type results from this. For predictions see figure 14.

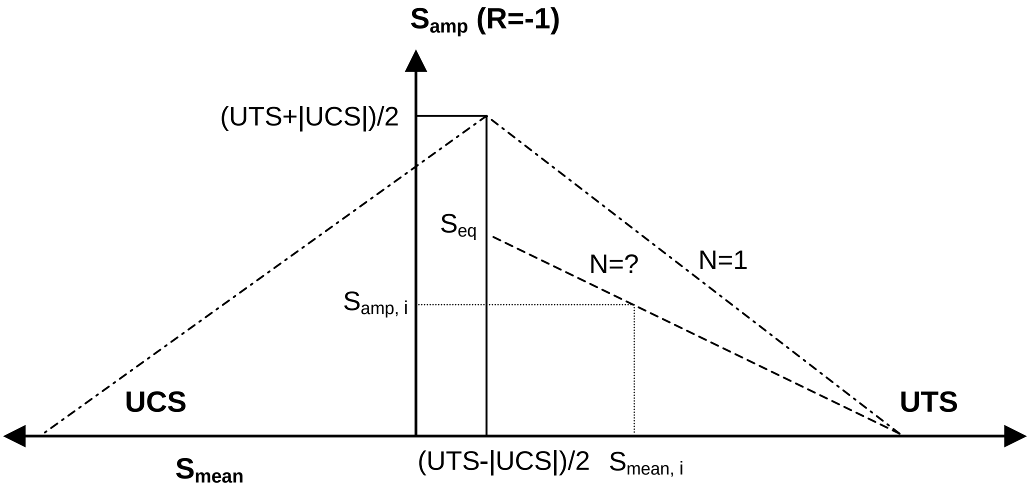

Figure 15 : ‘Shifted’ Linear Goodman Diagram

The GL-standards prescribe a CLD, which is very similar to the linear Goodman diagram, with the top of the CLD now located at a mean stress centered between ultimate tensile and ultimate compressive strength. Rewriting the GL-formulation to our current terminology and discarding any of the material constants, the lifetime for a certain mean-amplitude combination from this CLD is given by:

$$N = \left[\frac{UTS + |UCS| - |2 \cdot s_{mean} - UTS + |UCS||}{2 \cdot s_{amp}} \right]^{\frac{-1}{d}} \qquad (10)$$

The height of the CLD is now located at the average of UTS and |UCS|. A schematic for this CLD is given in figure 15. The corresponding prediction for WISPERX is given in figure 16. For this prediction, the UTS and UCS mentioned for OB at R=-1 have been used, as well as the slope parameter *d* from Table II. Note, that all of GL's partial safety-, static- and fatigue reduction factors, have been discarded. As a result, this prediction is non-conservative by approximately a decade, but it is appropriate for comparison of the prediction method. As is clear from the figure, the predictions from UP and from WMC give identical results.

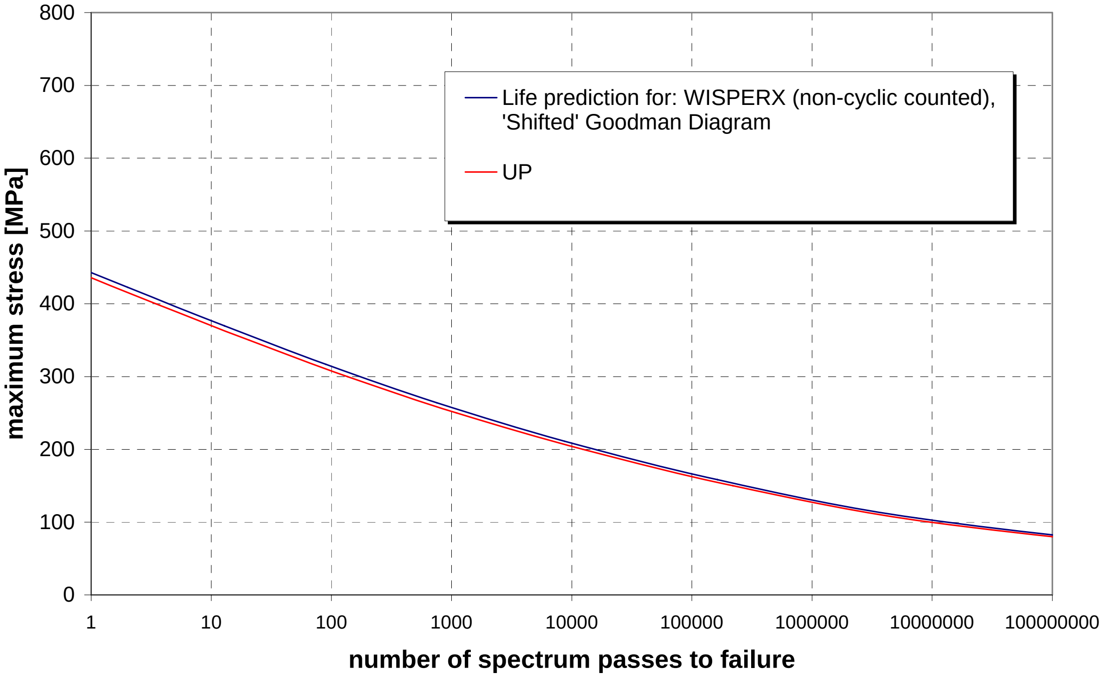

Figure 16: Lifeprediction using 'Shifted' Goodman diagram

When more experimental data are available for the material (i.e. for more R-ratios), use should be made in the lifetime prediction of this additional material characterisation. In the OPTIMAT BLADES project, S-N curves for R-ratios of 0.1 and 10 are also available, so a slightly more detailed CLD can be constructed, see figure 17. Strictly speaking, this is no longer a 'Goodman' diagram, so here it will be referred to as 'bi-linear Goodman type diagram'. This is because the leftmost and rightmost parts of the diagram (in region '1') are reminiscent of the classical Goodman diagram and similar formulations are derived to find an equivalent stress amplitude:

$$s_{eq,R_{right}} = \frac{s_{amp} \cdot UTS}{UTS - s_{mean} + s_{amp} \cdot \left(\frac{1+R_{right}}{1-R_{right}} \right)}$$, for s_{mean}>0, and (11a)

$$s_{eq,Rleft} = \frac{s_{amp} \cdot UCS}{UCS - s_{mean} + s_{amp} \cdot \left(\frac{1+R_{left}}{1-R_{left}} \right)}$$, for $s_{mean}<0$ (11b)

Here, the stress amplitude applies to the leftmost, or rightmost R-ratio, i.e. the first R-ratio that is found when travelling from the R=1 line (the s_{mean} –axis) to the R=-1 line (the s_{amp}-axis). This R-ratio is used to find the allowable number of cycles to failure in this region of the CLD.
How to find allowable number of cycles to failure in the remaining parts of the CLD (regions ‘2’) depends on the type of S-N definition that was chosen. E.g. for a lin-log S-N curve, an expression can be derived which gives the number of cycles explicitly; for the log-log type S-N curves, the number of cycles needs to be found using an iteration routine. The formulations to do this can be found in [12], and have been reproduced in a slightly different nomenclature in Appendix III. These are the formulations used by WMC, the appendix also contains calculation methods used by UP and DLR.

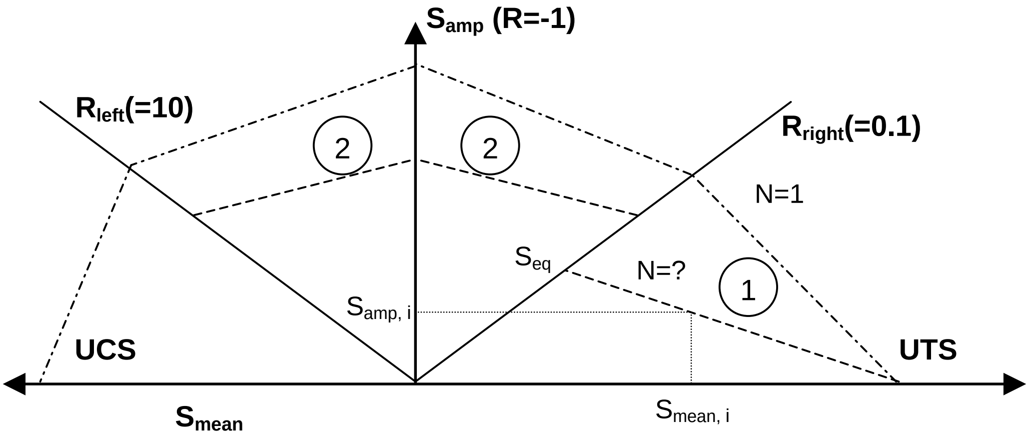

Figure 17 : Bi-Linear Goodman type Diagram

Lifetime predictions for cyclic counted WISPER and WISPERX, using a log-log and Sendeckyj S-N curve excluding static data, and a Bi-linear Goodman diagram are found in Figs. 18 and 19. Comparing figures 14, 16, and 18 (or 19), the bi-linear Goodman diagram gives the most conservative results, followed by the ‘shifted’ Goodman diagram and the Linear Goodman Diagram. Results across labs agree fairly well, any differences are likely be attributable to numerical differences. There is no large difference in predicted lifetime for WISPER and WISPERX. In terms of design-guidelines, and for this particular case, the shifted CLD-definition seems a reasonable compromise between realism and calculation effort: the calculations are simpler relative to the bi-linear- or full CLD, and less experimental effort is needed for the CLD definition. However, it must be pointed out that it is always more accurate to have a more detailed CLD definition.

Note, that there is a multitude of possible combinations of Rainflow counting algorithm (cyclic or non-cyclic), S-N curve definition (including, or excluding static data; using a log-log, lin-log, or other model), CLD-definition (Linear Goodman diagram, Bi-(or multi-)linear Goodman type diagram, a shifted Goodman diagram, etc.), of which only a few have been reported here.

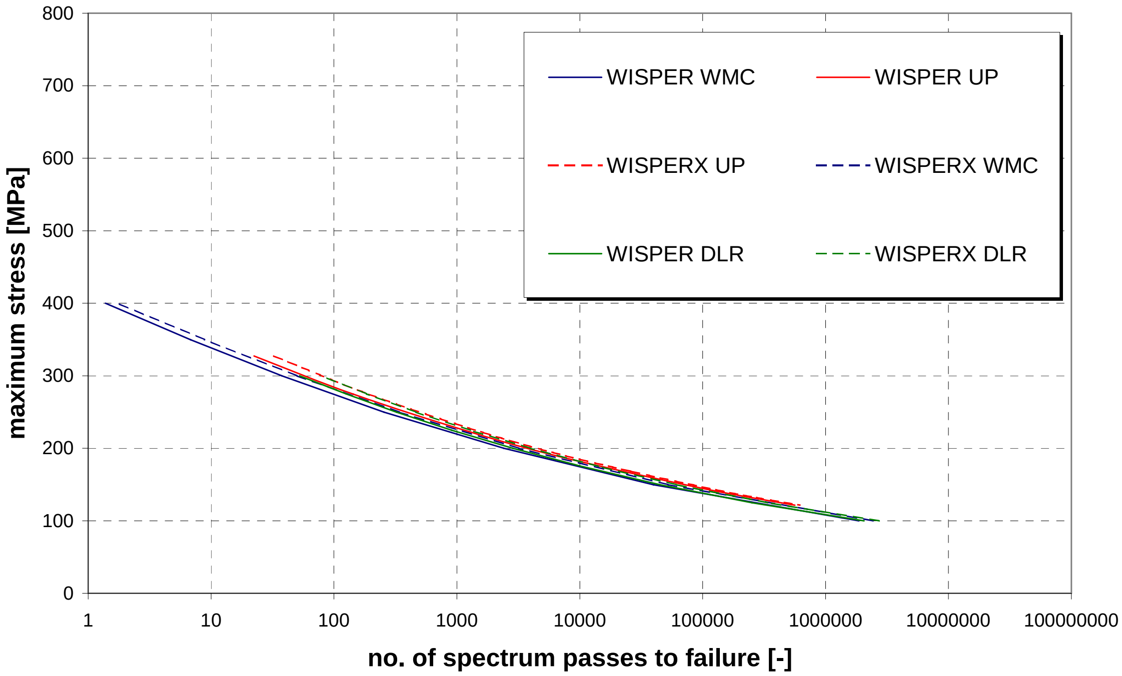

Figure 18: Prediction using bi-linear Goodman diagram and log-log S-N curves

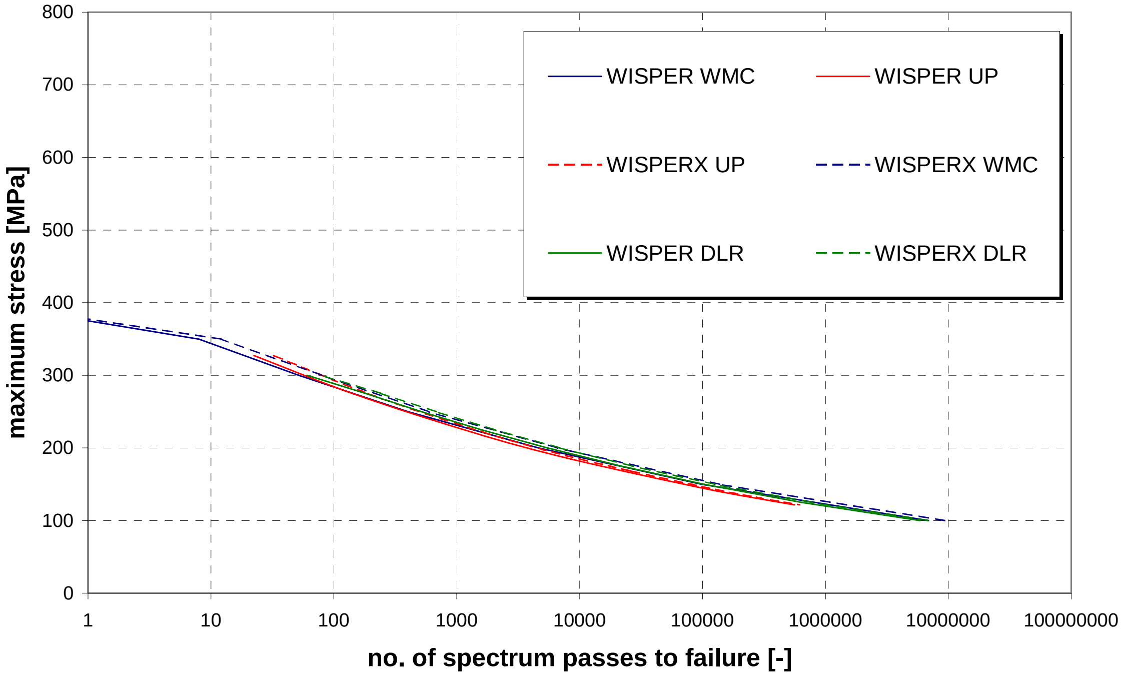

Figure 19: Prediction using bi-linear Goodman diagram and mean Sendeckyj S-N curves, Reliability P(N)=0.5

Concluding Remarks

From the results on identical datasets, it can be concluded that the results from the different labs compare favourably:

- Rainflow count matrices are identical, once the choice for a 'cyclic' or 'non-cyclic' algorithm is established
- Linear regression and finding the log-log S-N curve using 'Whitney's' procedure also yielded identical results
- The approach suggested by Sendeckyj can lead to different values of especially the C parameter when comparing results between laboratories. This depends on the type and settings of the optimization algorithms used in parameter determination. The shape-parameter and the slope at longer lifetime of the S-N curve turned out to be virtually identical, in all investigated datasets
- As for the lifetime predictions, three different Constant Life Diagrams were used. When using a Linear Goodman diagram and a log-log S-N curve at R=-1, the results between laboratories are identical. Also, a shifted and a bi-linear Goodman diagram was used, and here also, the results are very close. Any differences may be attributable to numerical differences

Now that the labs have shown that their algorithms are capable of producing the comparable results given the input is equal, some remarks on the methods to describe S-N curves for use in lifetime predictions are in order.

A suitable method for describing S-N curves for use in a lifetime prediction is hard to find. The disadvantage of the lin-log and log-log regression is, that they cannot accurately describe both static and fatigue data. This may lead to inaccurate lifetime predictions. On the other hand, the question whether static data are relevant in a fatigue life prediction has not been solved by performing a small number of high-strain rate static tests in a pilot project. Note, that in the final stage of the OPTIMAT programme, it was unanimously agreed that static data should only be included in an S-N diagram for informative purposes [13].

Due to its 'S-shape', the best candidate S-N curve, which does allow for a description over the entire range of experimental data, is the wearout model that was described in the discussion on Sendeckyj's method. Although finding the correct wearout parameters is slightly more tedious than performing a linear regression or even Whitney's method, the wearout model can be used in lifetime predictions with similar ease as a log-log S-N-curve. However, finding the correct wearout parameters is sensitive to the accuracy of the maximization routines in the algorithm. Also, the shape of the S-N curve is very sensitive to the value of especially the C-parameter. For some datasets there seems to be a strong tendency to allow a large range of C-values, due to the flatness of the C-alpha-curve in the region of maximum alpha.

Depending on what type of regression is chosen, and what type of S-N curve, the linear regression method and Whitney's method can be inherently the same. The question can be raised if the additional computational effort involved in the latter method is justified. However, this is a question of whether or not to use Weibull or Normal statistics and therefore lies outside the scope of this document.

In view of these remarks, some thought should be given to the following questions.

How important is the S-N description in the low-cycle region?

If the answer is 'not at all', then Sendeckyj's method becomes less interesting and a choice should be made between linear regression (excluding static data) and Whitney's method.

If the answer should be ‘very’, then additional research should be devoted to finding an S-N description that yields curves of the wear-out type, but preferably much less sensitive to numerical boundary conditions, and less sensitive to its parameters.
Part of this fundamental question, whether static data belong in an S-N diagram, and whether low cycle fatigue is of interest for long-life predictions, was addressed to a limited extent in this report. Another feature of Sendeckyj’s model however, viz. its capability to deal with residual strength data, was not.

What distribution function is most appropriate?

The linear regression method implicitly presupposes a (log)Normal distribution of the coupon lifetimes, whereas Whitney and Sendeckyj are Weibull-oriented. For mean S-N curves, the question of which S-N definition to use is irrelevant, but if lifetime predictions should be made using tolerance bounds or for a particular reliability, treatment and results will be different.

The abovementioned issues are not trivial and may not be resolved in the course of this project. Therefore, in future reports where S-N definitions are used, it is advisable to *at least* include an S-N curve from linear regression excluding static data. Other S-N curves can then be used at will (with proper reference to the conditions applied to the calculation), but it will be easier to find possible differences in the dataset by running a quick comparison of the linear regression curve.

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Appendix I: Sample datasets used for S-N curve definition

OB data									ECN data			
R=-1					R=0.1		R=10		R=0.1		R=-1	
Smax [MPa]	(TRANS-FORMED)	Life [no. of cycles]	Smax [MPa]	Life [no. of cycles]	Smax [MPa]	Life [no. of cycles]	Smax [MPa]	Life [no. of cycles]	Smax [MPa]	Life [no. of cycles]	Smax [MPa]	Life [no. of cycles]
103.82	104	6728478	492.6	1	160.0	1524977	277.86	876261	110	57730636	75.0	23532924
117.83	118	2719725	480.0	1	160.0	5167411	297.73	27302	110.0	33593418	115.0	430305
117.99	118	1623268	539.0	1	196.0	536659	293.64	53888	180.0	251697	115.0	501860
124.39	124	477127	517.1	1	196.0	1023212	339.42	1986	180.0	366524	115.0	503105
134.98	135	360314	474.0	1	196.0	1550777	269	735964	180.0	479884	115.0	536305
134.99	135	78055	504.3	1	196.0	1529500	294.12	20876	180.0	487314	115.0	972255
134.99	135	83510	508.4	1	196.0	472004	337.14	1540	180.0	652413	115.0	1385500
146.43	148	2091	474.4	1	196.0	569820	264.98	4744812	250.0	3006	210.0	450
148.49	148	48942	500.9	1	196.0	389441	325	2303	250.0	4806	210.0	517
150.00	148	1534	500.3	1	196.0	361994	274.99	231071	250.0	6056	210.0	1395
175.00	176	59468	498.7	1	260.0	42577	299.99	16743	250.0	6462	210.0	1403
177.49	176	16291	457.1	1	260.0	57647	299.97	4206	250.0	7555	210.0	1421
177.75	176	2608	522.0	1	260.0	71242	350	917	250.0	8855	210.0	3701
184.98	185	57038	504.5	1	260.0	36837	249.98	10173865	250.0	9255	210.0	3715
184.98	185	2098460	514.5	1	390.0	1177			250.0	10005	280.0	48
185.03	185	109901	508.1	1	390.0	1028			325.0	245	280.0	64
200.00	200	735186	504.4	1	390.0	995			325.0	471	280.0	66
224.98	224	655532	513.6	1	522.0	1			325.0	820	280.0	78
234.12	234	481189	520.8	1	504.5	1			325.0	844	280.0	92
250.01	250	2074	520.3	1	514.5	1			325.0	898		
250.01	250	8234	511.2	1	508.1	1			369	1		
250.03	250	637851	525.9	1	504.4	1			370	1		
490.6		1	549.4	1	513.6	1			371	1		
470.1		1	534.5	1	520.8	1			411	1		
452.4		1	547.4	1	520.3	1			437	1		
429.5		1	536.5	1	511.2	1						
402.8		1	550.2	1	525.9	1						
442.3		1	523	1	549.4	1						
443.8		1	522.9	1	534.5	1						
431.2		1	522.3	1	547.4	1						
452.8		1	517.6	1	536.5	1						
446.5		1	509.3	1	550.2	1						
446		1	497.5	1	523	1						
484.0		1	547.7	1	522.9	1						
472.2		1	551.5	1	522.3	1						
465.0		1	574.5	1	517.6	1						
481.0		1	578.0	1	509.3	1						
473.5		1	543.9	1	497.5	1						
469.5		1	525.9	1	547.7	1						
470.3		1	536.3	1	551.5	1						
458.0		1	539.2	1	574.5	1						
483.6		1	535.2	1	578.0	1						
467.8		1	528.8	1	543.9	1						
459.1		1	530.8	1	525.9	1						
			500.5	1	536.3	1						
			525.8	1	539.2	1						
			519.5	1	535.2	1						
			517.9	1	528.8	1						
					530.8	1						
					500.5	1						
					525.8	1						
					519.5	1						
					517.9	1						

Used as run-out in Whitney, not used in Sendeckvi

Not used in Whitney

UCS=462

UTS=528

Appendix II: Sendeckyj’s sample data, and results using WMC algorithm.

Data from Sendeckyj's paper

nr.	ID	stress [MPa]	residual strength [MPa]	life	test type
1	22aa6	2082		1	ST
2	22aa4	2048		1	ST
3	22ab5	2020		1	ST
4	22ab3	1979		1	ST
5	22aa1	1331		153	CA
6	22aa12	1289		267	CA
7	22ab6	1296		319	CA
8	22ab7	1334		436	CA
9	22aa14	965		1630	CA
10	22aa9	965		1330	CA
11	22ab10	965		1760	CA
12	22aa10	965		1220	CA
13	22aa11	758		10200	CA
14	22aa2	758		9000	CA
15	22ab9	758		7290	CA
16	22ab12	758		6750	CA
17	22aa5	586		74250	CA
18	22aa2	586		67490	CA
19	22ab9	586		36210	CA
20	22ab12	586		49800	CA
21	22aa13	483		138180	CA
22	22ab11	483		93880	CA
23	22ab4	483		224630	CA
24	22ab14	483		55780	CA
25	22aa7	379	1165	1122310	CA
26	22aa7	379	1979	213960	CA
27	22ab13	379		464810	CA
28	22ab2	379	1751	211800	CA

Parameter	Sendeckyj’s solution	WMC’s solution
S	0.157	0.15615
C	0.0485	0.050999
Shape parameter	19.54	20.13615
Scale parameter	2015 MPa	2015.212

Appendix III: Computation algorithms to find lifetime from a CLD, used by participants

Algorithms were delivered in their own nomenclature. For easier reading, this has been synchronized. The current formulations suggest stress-based calculations, however, DLRs original terminology used ε instead of σ; the methods are equally applicable to strains or stresses.

ALGORITHM USED BY WMC

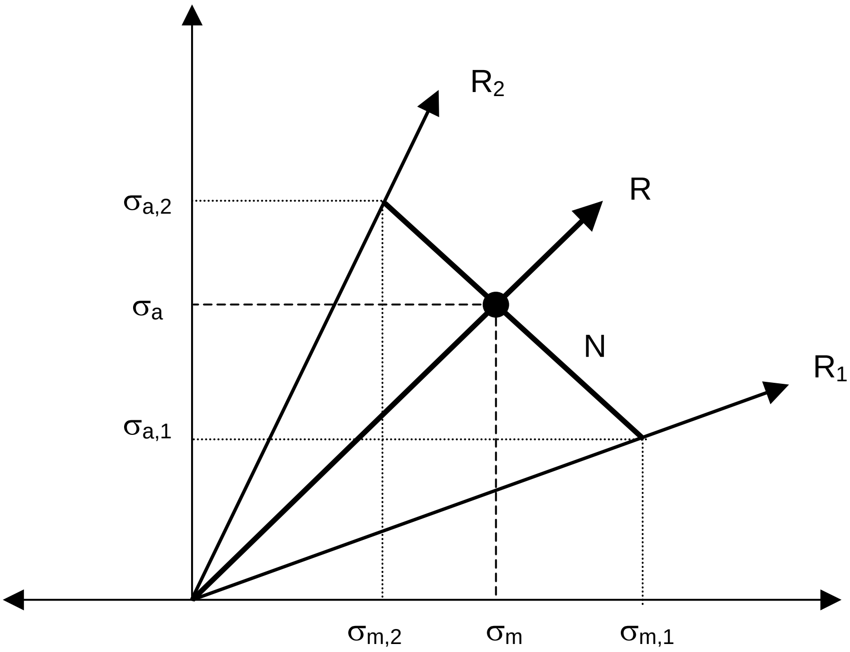

Figure A-1: Definition of terminology

From the geometry of figure A-1:

$$\frac{\sigma_{a,2}-\sigma_{a,1}}{\sigma_{m,1}-\sigma_{m,2}}=\frac{\sigma_a-\sigma_{a,1}}{\sigma_{m,1}-\sigma_m} \qquad \text{(A-1)}$$

By definition:

$$R=\frac{\sigma_m-\sigma_a}{\sigma_m+\sigma_a}\text{, hence} \qquad \text{(A-2a)}$$

$$\sigma_m=\sigma_a\frac{1+R}{1-R}=\sigma_a\cdot r\text{, and} \qquad \text{(A-2b)}$$

$$\sigma_a=\sigma_m\frac{1-R}{1+R}=\sigma_m\cdot\frac{1}{r} \qquad \text{(A-2c)}$$

Furthermore, the lin-log, log-log, and Sendeckyj S-N curve definitions:

$$\sigma_a=E+p\log(N) \qquad \text{(A-3)}$$

$$\sigma_a = FN^q \tag{A-4}$$

re-writing A-1:

$$\sigma_{a,2}\sigma_{a,1}r_1 - \sigma_{a,2}\sigma_a r + \sigma_{a,1}\sigma_a r = \sigma_{a,1}r_1\sigma_a - \sigma_{a,2}r_2\sigma_a + \sigma_{a,2}r_2\sigma_{a,1} \tag{A-5}$$

Combining A-4 and A-5, and rearranging leads to the following equation for the log-log S-N definition:

$$\left(F_1F_2r_1 - F_1F_2r_2\right)N^{q_2+q_1} + N^{q_2}\left(F_2\sigma_a r_2 - F_2\sigma_a r\right) + \left(F_1\sigma_a r - F_1\sigma_a r_1\right)N^{q_1} = 0 \tag{A-6}$$

The terms between brackets are all known, as well as the exponents. This equation cannot be solved analytically. Instead, a secant type iteration scheme can be used, which typically converges within ~15 iterations.

Combining A-3 and A-5, and rearranging terms leads to an expression for the lin-log S-N definition:

$$\begin{aligned}&\log(N)^2\left(r_1p_1p_2 - r_2p_1p_2\right) + \\ &\log(N)\left(E_2p_1r_1 + E_1p_2r_1 - E_2p_1r_2 - E_1p_2r_2 - \sigma_a rp_2 + \sigma_a rp_1 - \sigma_{ar}r_1p_1 + \sigma_a r_2p_2\right) + \\ &\left(E_2E_1r_1 - E_2E_1r_2 - E_2\sigma_a r + E_1\sigma_a r - E_1r_1\sigma_a + E_2r_2\sigma_a\right) = 0\end{aligned} \tag{A-7}$$

This quadratic equation can be solved analytically.

ALGORITHM USED BY UP

Philippidis et al. (2004)[1] have proposed a quite effective method, which treats the lin-log and log-log S-N curves in the same way, both without the need for iterations:

Combining A-1 and A-2b, and rewriting:

$$\sigma_a = \frac{\sigma_{a,1}\left(r_2 - r_1\right)}{\frac{\sigma_{a,1}}{\sigma_{a,2}}\left(r - r_1\right) + \left(r_2 - r\right)} \tag{A-8}$$

Then, substituting equations A-3 or A-4 for the lin-log or log-log S-N curves, and for different values of N, e.g. 10^3, 10^4, 10^5, 10^6, etc., stress amplitude – life-pairs are found, which are then subjected to linear regression in the lin-log or log-log space, yielding the parameters of the S-N curve corresponding to r:

$$\sigma_{a_r} = E_r + p_r \log(N) \tag{A-9}$$

$$\sigma_{a_r} = F_r N^{q_r} \tag{A-10}$$

---

[1] T. P. Philippidis et al., "Life prediction methodology for GFRP laminates under spectrum loading" Composites: Part A **35** (2004) 657-666

This equation is then used to find the value of N, corresponding to the given σ_a. The correlation coefficient of the linear regression is 1, so the solution is exact.

ALGORITHM USED BY DLR

The methods shown below were taken from Leylek (1997)[2], and are fundamentally equivalent to the algorithms described for WMC.

Rearranging A-1 with:

$$\sigma_a = \frac{1-R}{2} \cdot \sigma_{max} = A \cdot \sigma_{max} \qquad \text{(A-11a)}$$

$$\sigma_m = \frac{1+R}{2} \cdot \sigma_{max} = B \cdot \sigma_{max} \qquad \text{(A-11b)}$$

, and rearranging:

$$\sigma_{max,2} = \frac{\sigma_{max,1} C_a}{\left(C_b \sigma_{max,1} + C_c\right)} \qquad \text{(A-12)}$$

, where

$$C_a = B_1 \sigma_a - A_1 \sigma_m \qquad \text{(A-13a)}$$
$$C_b = A_2 B_1 - B_2 A_1 \qquad \text{(A-13b)}$$
$$C_c = -A_2 \sigma_m + B_2 \sigma_a \qquad \text{(A-13c)}$$

Sendeckyj's model

Wearout equation used to describe the two fatigue lines:

$$\sigma_{max,1} = \beta_1 \cdot \frac{\left(-\ln(P(N))\right)^{\frac{1}{\alpha_1}}}{\left(1-\left(1-C_1\right)N\right)^{S_1}} \cdot e^{\left(\frac{-U_\gamma (P(N))_1}{\sqrt{n_1} \cdot \alpha_1}\right)} \qquad \text{(A-14a)}$$

$$\sigma_{max,2} = \beta_2 \cdot \frac{\left(-\ln(P(N))\right)^{\frac{1}{\alpha_1}}}{\left(1-\left(1-C_2\right)N\right)^{S_2}} \cdot e^{\left(\frac{-U_\gamma (P(N))_2}{\sqrt{n_2} \cdot \alpha_2}\right)} \qquad \text{(A-14b)}$$

The parameters α and β are the shape and scale parameters of the Weibull distribution of the equivalent static strengths. the parameter n refers to the degrees of freedom, U_γ is the inverse of the Normal distribution.

---

[2] "Development of a computer code for estimating composite life using the Palmgren-Miner rule", Zafer Leylek, PhD Thesis, DLR IB 435-97/14.

Substituting these equations in A-12, and solving for N yields the mathematical solution. These equations are coupled and non-linear and an analytical solution not possible. A numerical procedure, comparable to the one used for A-6 is necessary.

Whitney's model

Description used to describe the two fatigue lines

$$N=\left(\frac{2F_1}{(1-R_1)\sigma_{\max,1}}\right)^{\frac{-1}{q_1}}\cdot\left(-\ln(P(N))\right)^{\frac{1}{\alpha_1}} \tag{A-15a}$$

$$N=\left(\frac{2F_2}{(1-R_2)\sigma_{\max,2}}\right)^{\frac{-1}{q_2}}\cdot\left(-\ln(P(N))\right)^{\frac{1}{\alpha_2}} \tag{A-15b}$$

Equating these (since N must be the same in A-15a and b):

$$\sigma_{\max,2}=\sigma_{\max,1}^{\frac{q_2}{q_1}}\cdot\left[\frac{4F_2F_1}{1-R_1-R_2+R_1R_2}\right]^{-\frac{q_2}{q_1}}\frac{\left(-\ln(P(N))\right)^{\frac{q_1}{\alpha_1}}}{\left(-\ln(P(N))\right)^{\frac{q_2}{\alpha_2}}}=\sigma_{\max,1}^{A}\cdot B \tag{A-16}$$

The constant life line can be found by combining A-12 and A-16, and solving for $\sigma_{\max,1}$ in:

$$\sigma_{\max,1}^{A+1}\cdot BC_b+\sigma_{\max,1}^{A}\cdot BC_c-\sigma_{\max,1}C_a=0 \tag{A-17}$$

This equation can be solved using the same numerical methods as can be used in A-6.

Linear regression

Description of the two fatigue lines:

$$N=\left(\frac{(1-R_1)\sigma_{\max,1}}{2F_1}\right)^{-q_1} \tag{A-18a}$$

$$N=\left(\frac{(1-R_2)\sigma_{\max,2}}{2F_2}\right)^{-q_2} \tag{A-18b}$$

A similar procedure as used in Whitney's method leads to the value of N.

Appendix IV: WISPER and WISPER X comparison between cyclic counted and non-cyclic counted algorithm; counting results for both spectra

WISPERX (non-cyclic counted)						WISPER (non-cyclic counted)						WISPERX (cyclic counted)						WISPER (cyclic counted)					
range	min	no. of halfcycles	range	min	no. of halfcycles	range	min	no. of halfcycles	range	min	no. of halfcycles	range	min	no. of halfcycles	range	min	no. of halfcycles	range	min	no. of halfcycles	range	min	no. of halfcycles
17	23	2	23	28	30	4	33	2	19	34	3112	17	23	2	23	21	98	4	33	2	19	28	4
17	24	12	23	29	332	5	36	92	19	36	4	17	24	12	23	22	2	5	36	92	19	29	88
17	28	22	23	30	90	7	36	22	20	22	2	17	28	22	23	27	2	7	36	22	19	33	2
17	29	3758	23	33	2	8	33	4	20	26	8	17	29	3758	23	28	30	8	33	4	19	34	3112
17	30	2	23	34	4	8	36	4	20	28	2	17	30	2	23	29	332	8	36	4	19	36	4
17	31	6	24	26	4	8	38	2	20	29	10	17	31	6	23	30	90	8	38	2	20	22	2
17	34	2104	24	27	440	9	36	48	20	31	1672	17	34	2104	23	33	2	9	36	48	20	26	8
17	36	6	24	28	120	9	37	2	20	34	120	17	36	6	23	34	4	9	37	2	20	28	2
17	37	2	24	29	14	10	31	18	20	35	636	17	37	2	24	26	4	10	31	18	20	29	10
18	22	2	24	31	4	10	36	156	21	28	2	18	22	2	24	27	440	10	36	156	20	31	1672
18	23	54	24	33	4	10	38	6	21	30	22	18	23	54	24	28	120	10	38	6	20	34	120
18	24	2	25	26	8	11	29	20	21	31	54	18	24	2	24	29	14	11	29	20	20	35	636
18	26	2542	25	27	4	11	33	2	21	34	778	18	26	2542	24	31	4	11	33	2	21	28	2
18	28	4	25	28	14	11	36	70628	21	36	1810	18	28	4	24	33	4	11	36	70628	21	30	22
18	29	4	25	30	2	12	25	48	22	26	10	18	29	4	25	26	8	12	25	48	21	31	54
18	30	8	25	31	2	12	28	2	22	28	36	18	30	8	25	27	4	12	28	2	21	34	778
18	33	5988	26	11	2	12	29	8706	22	29	62	18	33	5988	25	28	14	12	29	8706	21	36	1810
18	36	6	26	16	4	12	30	7690	22	30	48	18	36	6	25	30	2	12	30	7690	22	26	10
18	37	4	26	23	6	12	31	2	22	31	18	18	37	4	25	31	2	12	31	2	22	28	36
19	22	2	26	26	34	12	33	34	22	33	4	19	22	2	26	11	2	12	33	34	22	29	62
19	24	2	26	27	4	12	34	2	22	34	1086	19	24	2	26	16	4	12	34	2	22	30	48
19	27	4	26	29	34	13	27	32	23	21	98	19	27	4	26	23	6	13	27	32	22	31	18
19	28	4	26	31	60	13	28	2	23	22	2	19	28	4	26	26	34	13	28	2	22	33	4
19	29	88	27	26	4	13	29	2	23	27	2	19	29	88	26	27	4	13	29	2	22	34	1086
19	33	2	27	28	22	13	30	2330	23	28	30	19	33	2	26	29	34	13	30	2330	23	21	98
19	34	3112	28	10	2	13	33	44126	23	29	332	19	34	3112	26	31	60	13	33	44126	23	22	2
19	36	4	28	23	4	14	25	2	23	30	90	19	36	4	27	26	4	14	25	2	23	27	2
20	22	2	28	24	2	14	27	2	23	33	2	20	22	2	27	28	22	14	27	2	23	28	30
20	26	8	28	27	28	14	28	14	23	34	4	20	26	8	28	10	2	14	28	14	23	29	332
20	28	2	29	23	6	14	29	4	24	26	4	20	28	2	28	23	4	14	29	4	23	30	90
20	29	10	29	26	50	14	30	88	24	27	440	20	29	10	28	24	2	14	30	88	23	33	2
20	31	1672	29	27	2	14	31	19328	24	28	120	20	31	1672	28	27	28	14	31	19328	23	34	4
20	34	120	29	28	2	14	34	20824	24	29	14	20	34	120	29	23	6	14	34	20824	24	26	4
20	35	636	30	23	4	14	39	21860	24	31	4	20	35	636	29	26	50	14	39	21860	24	27	440
21	28	2	30	25	1	15	25	2	24	33	4	21	28	2	29	27	2	15	25	2	24	28	120
21	30	22	30	27	2	15	27	24	25	26	8	21	30	22	29	28	2	15	27	24	24	29	14
21	31	54	31	19	2	15	28	2	25	27	4	21	31	54	30	23	4	15	28	2	24	31	4
21	34	778	31	21	2	15	29	608	25	28	14	21	34	778	30	25	2	15	29	608	24	33	4
21	36	1810	31	24	6	15	30	12798	25	30	2	21	36	1810	30	27	2	15	30	12798	25	26	8
22	26	10	31	25	1	15	31	48	25	31	2	22	26	10	31	19	2	15	31	48	25	27	4
22	28	36	31	26	8	15	33	2184	26	11	2	22	28	36	31	21	2	15	33	2184	25	28	14
22	29	62	32	13	2	15	36	8	26	16	4	22	29	62	31	24	6	15	36	8	25	30	2
22	30	48	32	23	6	15	37	10624	26	23	6	22	30	48	31	26	8	15	37	10624	25	31	2
22	31	18	33	24	2	15	38	66	26	26	34	22	31	18	32	13	2	15	38	66	26	11	2
22	33	4	34	10	2	16	24	26	26	27	4	22	33	4	32	23	6	16	24	26	26	16	4
22	34	1086	34	12	2	16	27	2	26	29	34	22	34	1086	33	24	2	16	27	2	26	23	6
23	21	98	34	21	2	16	28	100	26	31	60				34	10	2	16	28	100	26	26	34
23	22	2	34	23	4	16	29	14	27	26	4				34	12	2	16	29	14	26	27	4
23	27	2	35	11	4	16	30	92	27	28	22				34	21	2	16	30	92	26	29	34
			35	20	4	16	31	16	28	10	2				34	23	4	16	31	16	26	31	60
			35	21	1	16	33	4	28	23	4				35	11	4	16	33	4	27	26	4
			36	7	2	16	36	8878	28	24	2				35	20	4	16	36	8878	27	28	22
			36	19	2	16	37	2	28	27	28				35	21	2	16	37	2	28	10	2
			36	22	2	16	38	8158	29	23	6				36	7	2	16	38	8158	28	23	4
			38	19	2	17	23	2	29	26	50				36	19	2	17	23	2	28	24	2
			39	12	2	17	24	12	29	27	2				36	22	2	17	24	12	28	27	28
			39	16	2	17	28	22	29	28	2				38	19	2	17	28	22	29	23	6
			40	15	2	17	29	3758	30	23	4				39	12	2	17	29	3758	29	26	50
			41	11	2	17	30	2	30	25	1				39	16	2	17	30	2	29	27	2
			41	14	2	17	31	6	30	27	2				40	15	2	17	31	6	29	28	2
			42	10	2	17	34	2104	31	19	2				41	11	2	17	34	2104	30	23	4
			43	12	2	17	36	6	31	21	2				41	14	2	17	36	6	30	25	2
			43	21	1	17	37	2	31	24	6				42	10	2	17	37	2	30	27	2
			45	10	4	18	22	2	31	25	1				43	12	2	18	22	2	31	19	2
			46	9	6	18	23	54	31	26	8				45	10	4	18	23	54	31	21	2
			48	7	2	18	24	2	32	13	2				46	9	6	18	24	2	31	24	6
			48	9	2	18	26	2542	32	23	6				48	7	2	18	26	2542	31	26	8
			54	1	1	18	28	4	33	24	2				48	9	2	18	28	4	32	13	2
			63	1	1	18	29	4	34	10	2				63	1	2	18	29	4	32	23	6
						18	30	8	34	12	2							18	30	8	33	24	2
						18	33	5988	34	21	2							18	33	5988	34	10	2
						18	36	6	34	23	4							18	36	6	34	12	2
						18	37	4	35	11	4							18	37	4	34	21	2
						19	22	2	35	20	4							19	22	2	34	23	4
						19	24	2	35	21	1							19	24	2	35	11	4
						19	27	4	36	7	2							19	27	4	35	20	4
						19	28	4	36	19	2										35	21	2
						19	29	88	36	22	2										36	7	2
						19	33	2	38	19	2										36	19	2
									39	12	2										36	22	2
									39	16	2										38	19	2
									40	15	2										39	12	2
									41	11	2										39	16	2
									41	14	2										40	15	2
									42	10	2										41	11	2
									43	12	2										41	14	2
									43	21	1										42	10	2
									45	10	4										43	12	2
									46	9	6										45	10	4
									48	7	2										46	9	6
									48	9	2										48	7	2
									54	1	1										48	9	2
									63	1	1										63	1	2

Table IV-1: From-to-matrix of cyclic counted WISPERX (insert shows differences for a non-cyclic counted signal)

Color legend

🟩	strictly compression cycles
🟨	strictly tension cycles
🟦	R=-1

	1	7	9	10	11	12	13	14	15	16	19	20	21	22	23	24	25	26	27	28	29	30	31	33	34	35	36	37	38	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	64
1																																																	1
7																																	1												1				
9																																													3		1		
10																													1					1								1			2				
11																												1								2						1							
12																																				1					1				1				
13																																			1														
14																																													1				
15																																													1				
16																																2													1				
19																																								1					1		1		
20																																													2				
21																																		49								1			1	1			
22																														1	1	1			1													1	
23																														1	27								3		2	3	2		3		2		
24																															6	1	1									1			3		1		
25																																													1				
26																																		1271		4		5		2	4	17	2		25		4		
27																																				2				1	220	2	2		14	1	1		
28																																			11	2	2	1	1	18	15	60	7		11		1		
29																																				1879	2	44	5		31	166	7		17				
30																																					1	4			11	24	45		1				
31																																						3			836	27	9		2	1	30		
33																																									2994	1			2	1	2		
34																																									1052		1556	60	389	543	2		
35																																													318				
36																																											3	3	2		905		
37					1																																							1	2				
38				1																																													
40														1	1																																		
41														1	27	6																																	
42										2				1		1																																	
43		1														1																																	
44				1									49					1271																															
45							1							1						11																													
46					2	1												4	2	2	1879																												
47																				2	2	1																											
48																		5		1	44	4	3																										
49															3					1	5																												
50											1							2	1	18																													
51						1									2			4	220	15	31	11	836	2994	1052																								
52				1	1								1		3	1		17	2	60	166	24	27	1																									
53															2			2	2	7	7	45	9		1556		3																						
54																									60		3	1																					
55		1	3	2		1		1	1	1	1	2	1		3	3	1	25	14	11	17	1	2	2	389	318	2	2																					
56													1						1				1	1	543																								
57			1								1				2	1		4	1	1			30	2	2		905																						
58														1																																			
64	1																																																

54	55	56	57	58	64
	1				
	1				
	3		1		
	2				
	1				
	1				
	1				
	1				
	1		1		
	2				
	1				1
				1	
	3		2		
	3		1		
		1			

Table IV-2: From-to-matrix of cyclic counted WISPER (insert shows differences for a non-cyclic counted signal)

	1	7	9	10	11	12	13	14	15	16	19	20	21	22	23	24	25	26	27	28	29	30	31	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	64
1																																																		1
7																																		1												1				
9																																														3		1		
10																													1						1								1			2				
11																												1									2						1							
12																																					1					1				1				
13																																				1														
14																																														1				
15																																														1				
16																																	2													1				
19																																									1					1		1		
20																																														2				
21																																			49								1			1	1			
22																															1	1	1			1													1	
23																															1	27								3		2	3	2		3		2		
24																															13	6	1	1									1			3		1		
25																												24		1	1															1				
26																																			1271		4		5		2	4	17	2		25		4		
27																															16	1	12	1			2				1	220	2	2		14	1	1		
28																															1	1	7	1	50	11	2	2	1	1	18	15	60	7		11		1		
29																															10	4353	1	2	304	7	1879	2	44	5		31	166	7		17				
30																																	3845	1165	44	6399	46	1	4			11	24	45		1				
31																																9		1		9664	24	8	3			836	27	9		2	1	30		
33																												1				2			1	17	22063		1092	2		2994	1			2	1	2		
34																																					1		10412			1052		1556	60	389	543	2		
35																																														318				
36																																46		11	2	24	78	35314				4	4439	3	3	2		905		
37					1												24							1													1						5312	1	1	2				
38				1																																	1		3					33	4079					
39																	1																											10930						
40														1	1	13	1		16	1	10																													
41														1	27	6			1	1	4353		9	2			46																							
42										2				1		1			12	7	1	3845																												
43		1														1			1	1	2	1165	1				11																							
44				1									49					1271		50	304	44		1			2																							
45							1							1						11	7	6399	9664	17			24																							
46					2	1												4	2	2	1879	46	24	22063	1		78	1	1																					
47																				2	2	1	8				35314																							
48																		5		1	44	4	3	1092	10412				3																					
49															3					1	5			2																										
50											1							2	1	18																														
51						1									2			4	220	15	31	11	836	2994	1052		4																							
52				1	1								1		3	1		17	2	60	166	24	27	1			4439	5312																						
53															2			2	2	7	7	45	9		1556		3	1	33	10930																				
54																									60		3	1	4079																					
55		1	3	2		1		1	1	1	1	2	1		3	3	1	25	14	11	17	1	2	2	389	318	2	2																						
56													1						1				1	1	543																									
57			1								1				2	1		4	1	1			30	2	2		905																							
58														1																																				
64	1																																																	

Color legend

🟩	strictly compression cycles
🟨	strictly tension cycles
🟦	R=-1

54	55	56	57	58	64
	1				
	1				
	3		1		
	2				
	1				
	1				
	1				
	1				
	1		1		
	2				
	1				1
				1	
	3		2		
	3		1		
		1			