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LIFE PREDICTIONS OF LONG FIBER COMPOSITES IN EXTREME ENVIRONMENTAL CONDITIONS USING DAMAGE EVOLUTION APPROACH

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ABSTRACT

The effects of extreme environmental conditions on damage evolution and lifetime of long fiber polymer based composites are studied experimentally and theoretically. The exposure to salt water is the extreme environmental conditions that are considered in this study. The continuum damage mechanics is considered as a promising approach to develop thermodynamically consistent formulation of constitutive law for media with damage, that would account for effects of temperature changes (non-isothermal conditions) and environmental (moisture) conditions as they affect the damage evolution. This is because the additional internal state variables (ISV), that would account for considered phenomenon, can be included. The thermodynamically consistent damage dependent lamination theory is used at this point of the study, to describe inelastic response to the applied load for a long fiber composite laminates at considered materials. In this study, the internal state variables that accounts for damage of UD composite in fiber direction are formulated within the terms of the used theory, and the function that describe the corresponding internal state variable are determined experimentally and calculated using macromechanics. The stiffness degradation is associated with corresponding damage evolution providing the tool for experimental determination of ISV accounting for corresponding damage. The theoretically calculated and experimentally determined functions of ISV are used to predict the lifetime of the laminates with arbitrary layup.

1 INTRODUCTION

Typical for reinforced composites are inelastic effects. The inelasticity is commonly associated with viscoelasticity, viscoplasticity and damage evolution. The damage evolution is a complicated phenomenon to account for and to characterize. It becomes even more complicated if the damage evolution at different environmental conditions is considered.

The objectives of this work is to study experimentally and theoretically the effect of extreme environmental conditions on damage evolution and lifetime of long fiber polymer based composites. The exposure to salt water are the extreme environmental conditions that are considered in this study. The ambient room conditions are selected as a reference environmental conditions with respect to which the mechanical properties of the considered materials at extreme conditions will be analyzed and compared. The material properties of the reference material and conditioned material (exposed to salt water) have been studied experimentally and theoretically.

The continuum damage mechanics is a promising approach to develop thermodynamically consistent formulation of constitutive law for media with damage. This would account for effects of temperature changes (non-isothermal conditions) and environmental (moisture) conditions. This is because the additional internal state variables (ISV) that would account for considered phenomenon

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can be included. The thermodynamically consistent damage dependent lamination theory, formulated by D.H. Allen [1,2], is used at this point of the study, to describe inelastic response to the applied load for a long fiber composite laminates at reference conditions (isothermal environmental conditions). In this study, the internal state variable that accounts for damage of UD composite in fiber direction is formulated within the terms of the applied theory, and the function that describe the corresponding internal state variable is determined and analyzed theoretically and experimentally. The stiffness degradation is associated with corresponding damage evolution providing the tool for experimental determination of ISV accounting for considered damage. The determined functions of ISV are used to predict the lifetime of laminates with arbitrary lay-ups.

2 THERMODYNAMIC CONSTRAINTS ON SYSTEM WITH LOCAL DAMAGE

The constitutive relationships for media with cracks has been developed by [1,2] and herein used in form as

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} + I^{\xi}_{ijkl}\alpha^{\xi}_{kl}, \tag{1}$$

where I^{ξ}_{ijkl} is damage dependent moduli, and $\xi = 1,...,N$ accounts for different damage modes. The locally averaged internal state variables associated with energy dissipation due to the cracking is defined as first proposed by [3,4] and utilized by [1,2] as

$$\alpha_{kl} = \frac{1}{V}\int_{S_2} u_k n_l dS_2 \tag{2}$$

where α_{kl} are components of internal state variable tensor, V is a local volume in which statistical homogeneity can be assumed, u_k and n_l are crack face displacement and normal respectively, S_2 is a crack surface area.

The reduction of the number of unknown constants in (1) can only be done by specifying the material symmetry and specific damage modes of interest. By utilizing the conditions of the symmetry of stress and strain tensors, leading to symmetry in $C_{ijkl} = C_{jikl}$, $I^{\xi}_{ijkl} = I^{\xi}_{jikl}$, and $C_{ijkl} = C_{ijlk}$ respectively, the number of unknown constants in (1) are reduced. Further, the symmetry conditions for compliance tensor, $C_{ijkl} = C_{klij}$ are provide by considering the second derivative of free energy for uncracked body. If the tensor of compliance is considered symmetric, $C_{ijkl} = C_{klij}$, this type of symmetry applies only to the compliance matrix. It is not required for damage tensor, I^{ξ}_{ijkl}. Material symmetry can be utilized for further simplification of constitutive equations, if the transverse isotropy is considered.

Further, the shape and size of damage tensor, I^{ξ}_{ijkl}, have to be determined for each damage mode ξ in particular. It is assumed that damage from loading in fiber direction induces orthotropy in three principle planes. Therefore, the damage tensor I^{ξ}_{ijkl}, is an orthotropic tensor with 15 unknown constants given in material symmetry axis, where $\xi = 1$ accounts for induced damage due to loading in x_1 principal direction (fiber direction), and $\xi = 2$ will be used to account for transverse matrix cracking.

The general constitutive relationships (1) can be written using the Voigt notation [1,2] as

$$\sigma_i = C_{ij}\varepsilon_j + I_{ik}^{\xi}\alpha_k^{\xi} \quad (3)$$

3 EQUATIONS FOR THE LAMINATE

If the constitutive relationships of single ply are given by (3), the constitutive relationships for general laminate with damage are obtained and written in the matrix form

$$\{N\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \{\sigma\} dz = [A]\{\varepsilon\} + [B]\{k\} + \{f^{(D)}\} + \{f^{(1)}\} + \{f^{(2)}\} + \{f^{(3)}\}, \quad (4)$$

where extensional stiffness matrix is defined as $[A] = \sum_{k=1}^{n} [Q]_k (z_k - z_{k-1})$, and the coupling stiffness matrix of the laminate is given as $[B] = \frac{1}{2}\sum_{k=1}^{n}[Q]\left(z_k^2 - z_{k-1}^2\right)$. The different forms of damage that are accounted for within the (4), are represented by $\{f^{(D)}\}$, $\{f^{(1)}\}$, $\{f^{(2)}\}$, and $\{f^{(3)}\}$, corresponding to delamination, fiber fracture, transverse cracking and shear debonding respectively. The fiber fracture that takes place due to the loading in fiber direction is the only damage mode that will be addressed within this study. Therefore, the details regarding the $\{f^{(D)}\}$, $\{f^{(2)}\}$, and $\{f^{(3)}\}$ are left out of the scope of this paper. The fiber fracture that takes place due to the loading in fiber direction is described by

$$\{f^{(1)}\} = \sum_{k=1}^{n} \left[I^{(1)}\right]_k (z_k - z_{k-1}) \{\alpha^{(1)}\}_k , \quad (5)$$

where $\{f^{(1)}\}_k$ is a damage stiffness tensor of k^{th} layer, and $\{\alpha^{(1)}\}_k$ is internal state variable accounting for strain perturbations due to the considered damage. Generally, $\{\alpha^{(1)}\}_k$ can be determined experimentally, or calculated using micromechanics approach. The measurements of stiffness degradation of specially designed UD laminate must be used in order to determine $\{\alpha^{(1)}\}_k$ experimentally.

4 DETERMINATION OF DAMAGE TENSOR, $\{\alpha^{(1)}\}_k$

The expected damage due to static loading in fiber direction is schematically illustrated in Figure 1.

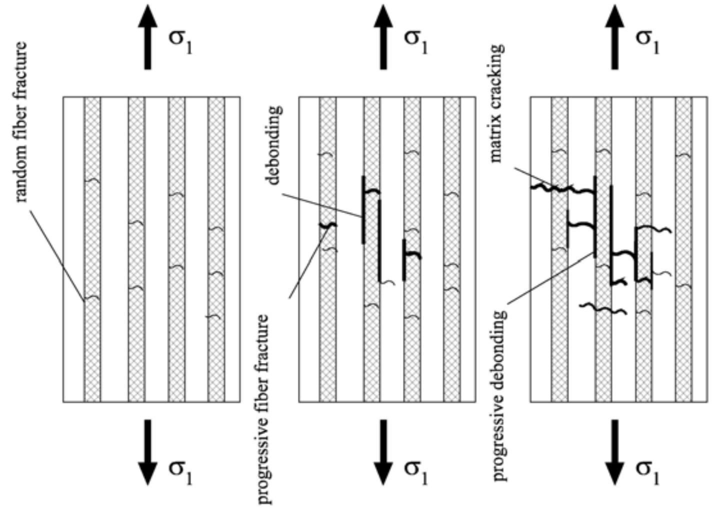

Figure 1. Damage development in composite laminate loaded in fiber direction.

The progressive fiber fracture, combined with debonding and matrix cracking at higher applied strain levels, can be described by representative unit element as illustrated in Figure 2. The considered unit element allows also to account for the debonding effects and matrix cracking effects on the crack opening displacement.

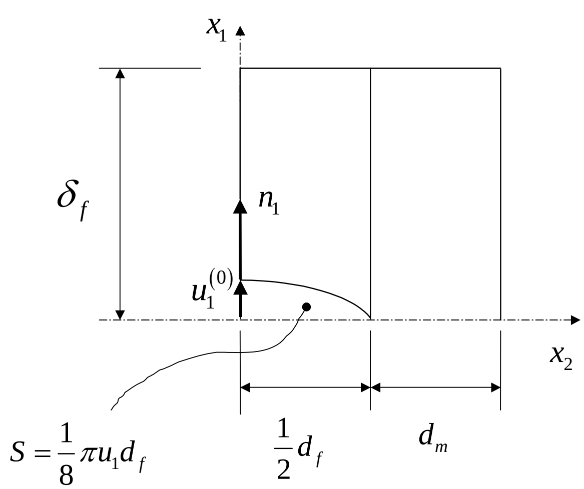

Figure 2. Representative volume element. δ_f *is ineffective length,* d_f *is diameter of the fiber,* d_m *is dimensions of the surrounding matrix, and* S *is ¼ of the area of the ellipse under the crack profile line . The dimensions are calculated based on the fiber packing in hexagonal array.*

According to the general definition, the damage tensor is described in terms of crack opening displacement as

$$\alpha_{kl}^{(1)} = \frac{1}{V}\int_{S_2} u_k n_l dS_2 \ , \tag{6}$$

where S_2 is the total crack surface area of the all cracks in the representative volume, n_l and u_k are normal to the crack surface and the crack surface displacement. The integral in (6) can be written as

$$\alpha_{kl} = \frac{1}{V}\sum_{m=1}^{M}\int_{S_2^{(0)}} u_k n_l dS_2^{(0)} , \tag{7}$$

where $S_2^{(0)}$ is crack surface area of single crack. If the crack does not change the orientation, see Figure 2, the normal to crack surface have only one component, $n_1 = 1$, $n_2 = n_3 = 0$. It leads to the,

$$\alpha_{kl}^{(1)} = \left\{\alpha_{11}^{(1)} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \alpha_{31}^{(1)} \quad 0 \quad \alpha_{21}^{(1)}\right\}^T \tag{8}$$

where the $\alpha_{11}^{(1)}$ accounts for crack opening in Mode-I, and according to (7) can be expressed in form of crack opening displacement and crack density function as

$$\alpha_{11}^{(1)} = \frac{1}{V}\sum_{m=1}^{M}\int_{S_2^{(0)}} u_1 dS_2^{(0)} = \frac{1}{V}\sum_{m=1}^{M}\int_{x_2} u_1 dx_2 \tag{9}$$

Considering the finite size of the arbitrary representative volume, $V = dx_1 dx_2 dx_3$, where $dx_1 = \delta_f$ and $dx_2 = \frac{1}{2}d_f + d_m$, see Figure 2, also considering two dimensional case with unit thickness, $dx_3 = 1$

$$\alpha_{11}^{(1)} = \frac{1}{k\delta_f d_f \frac{1}{\sqrt{V_f}}}\sum_{m=1}^{M}\int_{x_2} u_1 dx_2 , \tag{10}$$

or in form

$$\alpha_{11}^{(1)} = \frac{m}{k\delta_f d_f \frac{1}{\sqrt{V_f}}}\int_{x_2} u_1 dx_2 \Bigg|_{\int_{x_2} u_1 dx_2 = const\,\forall m} , \tag{11}$$

where $dx_2 = kd_f \frac{1}{\sqrt{V_f}}$, d_f is diameter of the fiber, V_f is fiber volume fraction, and $k = 0.8244$ is a generally proportionality coefficient. Both, k and dx_2 are calculated on bases of the hexagonal fiber packing. The integral part in the (11) gives the area under the crack surface profile, $S = \int_{x_2} u_1 dx_2 = \frac{1}{8}\pi u_1^{(0)} d_f$, where $u_1^{(0)}$ is the crack opening displacement at $x_2 = 0$, see Figure 2. Consequently (11) can be rewritten as

$$\alpha_{11}^{(1)} = \frac{1}{8}\frac{\pi u_1^{(0)} m\sqrt{V_f}}{k\delta_f}, \tag{12}$$

where $u_1^{(0)}$ is considered as a function of the applied strain, geometry and stiffness ration of fiber and matrix, $u_1^{(0)} = u_1^{(0)}\left(\varepsilon_1, d_f, E_f, E_m\right)$. The number of crack, m are calculated in unit cell, and will have limits $0 \le m \le 1$. The maximum crack density can be calculated by using the fact that one crack per ineffective length is possible

$$\rho = \frac{m}{\delta_f} = \rho_{\max}\big|_{m=1} = \frac{1}{\delta_f}. \tag{13}$$

Both ineffective length, δ_f, and number of cracks per unit fiber length as a function of applied strain, can be obtained experimentally from Single Fiber Fragmentation Test. However, The Weibull statistics for the fiber strength can be used to generate the crack density function, and in this case, FEM can be used to determine ineffective length. Also analytical expressions could be used to calculate δ_f. This approach will lead to overestimated $\alpha_{11}^{(1)}$ for large applied strain levels. It is because practically the crack density in composite would be expected somewhat less than theoretical, $\rho_{\max} = 1/\delta_f$. The stochastic methods, or Monte-Carlo simulations could give more realistic crack density function for the composite.

5 EQUATIONS FOR UNIDIRECTIONAL LAMINATE (UD)

The unidirectional laminate with fiber orientation in loading direction, and formulate the constitutive relationships for this particular case. In case of static loading, $\left\{N\right\} = \left\{N_x \quad 0 \quad 0\right\}^T$, there is only one type of damage expected, and it can be described with the internal state variable $\alpha^{(1)}$, see section. The constitutive relationships for the considered laminate can be written in a compact matrix form as

$$\left\{N\right\} = \left[A\right]\left\{\varepsilon\right\} + \left\{f^{(1)}\right\} \tag{14}$$

For UD laminate $\left[0_n\right]$, $\left[I^{(1)}\right]_k = \left[I^{(1)}\right]\forall k$, $\left[\alpha^{(1)}\right]_k = \left[\alpha^{(1)}\right]\forall k$ and $t_k = t\ \forall k$. It simplifies (14) to form

$$\left\{N\right\} = \left[A\right]\left\{\varepsilon\right\} + \left[I^{(1)}\right]\left\{\alpha^{(1)}\right\}h \tag{15}$$

Considering loading conditions in fiber direction, $\left\{N\right\} = \left\{N_1 \quad 0 \quad 0\right\}^T$, the first two equations of (15) are written as

$$N_1 = A_{11}\varepsilon_1 + A_{12}\varepsilon_2 + I_{11}^{(1)}\alpha_{11}^{(1)}h \tag{16}$$

$$0 = A_{21}\varepsilon_1 + A_{22}\varepsilon_2 + I_{21}^{(1)}\alpha_{11}^{(1)}h \tag{17}$$

We have now two equations with three unknowns in them. The number of unknowns could be reduced by using, $-C_{ijkl} = I_{ijkl}$ [1,2], and $A_{ij} = C_{ij}h$. The (16), and (17) are reduced to

$$N_1 = A_{11}\varepsilon_1 + A_{12}\varepsilon_2 + C_{11}\alpha_{11}^{(1)}h \tag{18}$$

where A_{ij}, and C_{ij} are elastic constants, that can be calculated using micromechanics or can be determined experimentally. $\alpha_{11}^{(1)}$ is the internal state variable that accounts for the damage that occur from static loading in fiber direction.

6 EXPERIMENTAL DETERMINATION OF $\alpha_{11}^{(1)}$

The stiffness degradation as a function of applied strain can be utilized in order to determine $\alpha_{11}^{(1)}$ experimentally. Introducing the definition of the stiffness

$$E_1(\varepsilon) \equiv \frac{1}{h}\frac{\partial N_1}{\partial \varepsilon_1}, \tag{19}$$

and using (16), (19) results into

$$E_1(\varepsilon_1) = \frac{1}{h}A_{11} + \frac{1}{h}\frac{\partial \alpha_{11}^{(1)}}{\partial \varepsilon_1}\left(\frac{A_{12}A_{21}}{A_{22}} - A_{11}\right) - \frac{1}{h}\frac{A_{12}A_{21}}{A_{22}}. \tag{20}$$

Further, the $\alpha_{11}^{(1)}$ is expressed from (20) as function of measured stiffness for particular applied strain level,

$$\alpha_{11}^{(1)}(\varepsilon_1) = \frac{h\int_{\varepsilon_1} E_1(\varepsilon_1)\,d\varepsilon_1 + \frac{A_{12}A_{21}}{A_{22}}\varepsilon_1 - A_{11}\varepsilon_1}{\left(\frac{A_{12}A_{21}}{A_{22}} - A_{11}\right)}. \tag{21}$$

7 RESULTS AND DISCUSSIONS

The stiffness degradation of a UD laminate, $[0_4]_T$, is measured experimentally using quasi-static tensile loading-unloading test. The same is done for both conditioned (exposed to salt water for 6 month at room temperature tested at ambient room temperature) and reference (non-conditioned, tested at ambient room temperature) materials. The whole hysteresis loop of the i^{th} loading-unloading cycle is used to calculate the Young's modulus, $E_1(\varepsilon_1)$, for corresponding applied strain. It was observed, that the stiffness degradation measured is consistent for all individual specimens.

The measured stiffness degradation is normalized, E_i / E_0 and described by a second order polynomial, see Figure 2. The same relative (normalized) stiffness degradation was observed for both conditioned and reference materials. However, the conditioned material has lover values for strain to failure, $\varepsilon_1 \approx 1.75[\%]$. In the tensile test there are limits of how close the maximum damage state can be characterized. In this work the damage state is characterized up to applied strain $\varepsilon_1 = 2.25[\%]$ for the reference material, and $\varepsilon_1 \approx 1.75[\%]$ for the conditioned material.

The rapid fracture process takes place within very short interval of applied strain after this point. The fracture that leads to the final failure is localized and would not represent the overall material property.

The obtained function for E_i / E_0 can be used into (21) in order to calculate $\alpha_{11}^{(1)}(\varepsilon_1)$. The experimentally determined $\alpha_{11}^{(1)}(\varepsilon_1)$ reflects the total effect of the multiple fracture mechanism that takes place. It would be difficult to separate the different mechanisms if the $\alpha_{11}^{(1)}(\varepsilon_1)$ is determined experimentally. The micromechanics approach is used (12) and (13) to calculate $\alpha_{11}^{(1)}(\varepsilon_1)$ accounting for fiber cracking and crack density assumptions. The FEM is used for calculating the crack opening displacement as function of applied strain, $u_1^{(0)}$ in (12). The Monte Carlo simulations, utilizing weakest link assumptions and stress global shearing mechanism, are used to produce the crack density function.

The experimentally determined and theoretically calculated values of $\alpha_{11}^{(1)}(\varepsilon_1)$ are given in Figure 3.

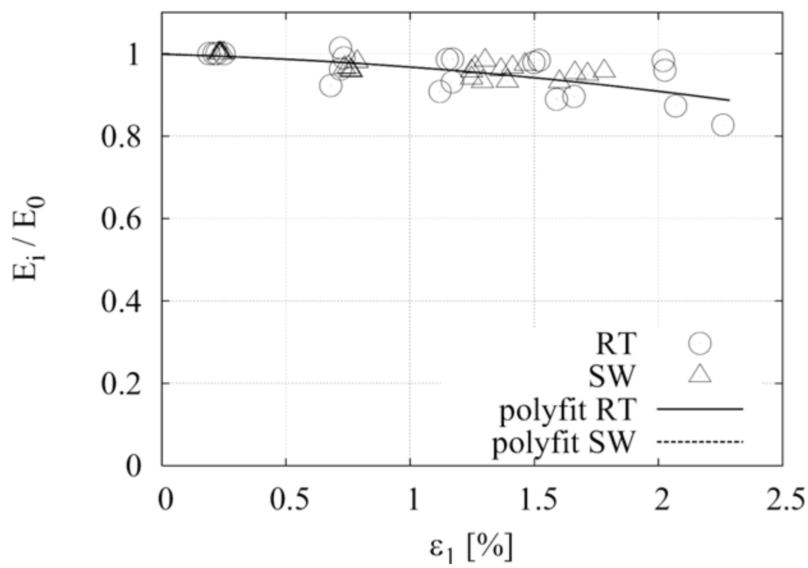

Figure 2: Stiffness degradation of UD composite. The second order polynomial, $y = a_2 x^2 + a_1 x + a_0$ *is used for approximation. The calculated constants are,* $a_0 = 1.0$, $a_2 = -1.76$ *and* $a_2 = -136.9$.

The theoretically predicted $\alpha_{11}^{(1)}(\varepsilon_1)$ is overestimated at higher applied strain. It is due to use of weakest link and global stress shearing assumptions as well as using the method of chain of bundles in Monte Carlo simulations. It leads to artificially increased number of cracks. The local stress shearing mechanism would give more realistic description of mechanism, and will be implemented in future. As well as the use of conditions of formation clusters of multiple fiber breaks would reduce the predicted number of cracks. It is believed that the accounting for the debonding and using the local load sharing mechanisms would capture the difference of the crack saturation level at different applied strain levels for compared materials. Also the maximum crack density should be controlled by the number of cracks at strain level where the further increase of the applied strain would lied to ultimate failure of the material in the considered volume.

In order to validate the outlined approach and the measured $\alpha_{11}^{(1)}(\varepsilon_1)$, the strain-stress curve of independent tensile test of UD laminates, $[0_4]_T$, can be predicted by using the obtained function of $\alpha_{11}^{(1)}(\varepsilon_1)$ into the constitutive equations for laminate, (4). The results of predictions are given in Figure 4.

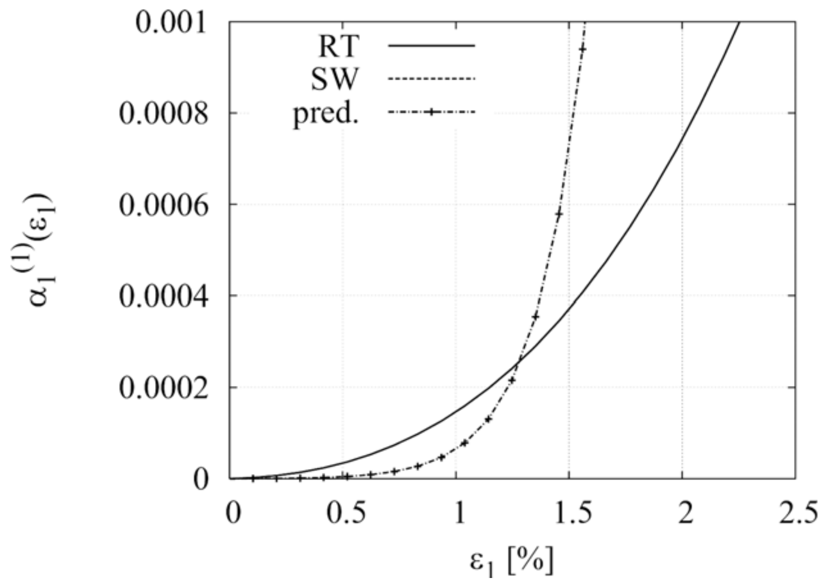

Figure 3: Calculated ISV $\alpha_{11}^{(1)}(\varepsilon_1)$, RT-reference material tested at ambient room conditions, SW-the same material that has been exposed to the salt water for 6month (tested at ambient room conditions).

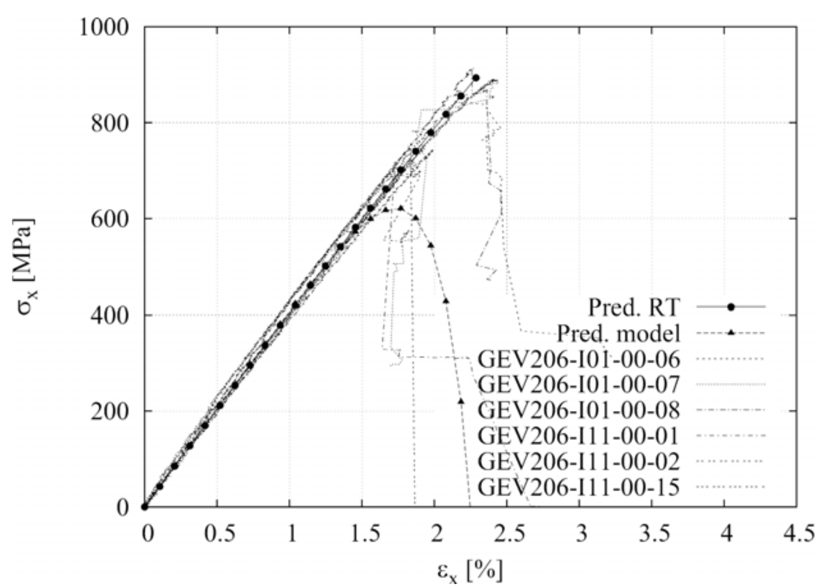

Figure 4: Predictions of strain-stress behavior of UD laminates in tensile test compared with experimental data, Pred. RT – predictions using experimentally measured $\alpha_{11}^{(1)}(\varepsilon_1)$ for reference conditions, Pred. model- predictions using calculated (predicted) $\alpha_{11}^{(1)}(\varepsilon_1)$.

Considering the limits of the damage evolution characterization, there is a considerable good agreement between the predicted strain and stress values and experimental data. Since the loading conditions, $\{N\}=\{N_1 \quad 0 \quad 0\}^T$, are applied for the test, the inverse constitutive equations must be used in order to predict strain-stress curve. The inverse constitutive relationships in general are given in [5].

8 CONCLUDING REMARKS

The continuum damage mechanics approach is used to account for and characterize the damage evolution in long fiber laminated composites. The internal state variable evolution law associated with damage in fiber direction is formulated in terms of used theory, and determined experimentally and theoretically. The theoretically and experimentally determined evolution law of internal state variable are used in damage dependent constitutive relationships for laminate in order to predict the static strain - stress behavior of arbitrary laminates.

The measurements of the stiffness degradation are used to determine the corresponding internal state variable function. The relationships between particular ISV and corresponding stiffness degradation is formulated for considered damage mechanism.

It was observed experimentally that the normalized stiffness degradation of conditioned and reference materials are rather close, and are approximated with the same polynomial. The progressive fiber fracture is considered as main contributing mechanism. The outlined micromechanics methods gives possibilities to study the interface strength and debond crack growth effects on formulated damage tensor.

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