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Abstract

The effects of extreme environmental conditions on damage evolution and lifetime of long fiber polymer based composites are studied experimentally and theoretically. Altered temperature and exposure to a salt water are the extreme environmental conditions that are considered in this study. The continuum damage mechanics is considered as a promising approach to develop thermodynamically consistent formulation of constitutive law for media with damage, that would account for effects of temperature changes (non-isothermal conditions) and environmental (moisture) conditions as they affect the damage evolution. This is because the additional internal state variables (ISV), that would account for considered phenomenon, can be included. The thermodynamically consistent damage dependent lamination theory is used at this point of the study, to describe inelastic response to the applied load for a long fiber composite laminates at reference environmental conditions (ambient room conditions). In this study, the internal state variables that accounts for damage of UD composite in fiber direction are formulated within the terms of the theory, and the function that describe the corresponding internal state variable are determined experimentally. The ISV evolution law associated with damage in fiber direction are additionally formulated within the terms of the model. The stiffness degradation is associated with corresponding damage evolution providing the tool for experimental determination of ISV accounting for corresponding damage. The experimentally determined functions of ISV are used to predict the lifetime of the laminates with arbitrary layup.

Introduction

Typical for reinforced composites are inelastic effects. The inelasticity is commonly associated with viscoelasticity, viscoplasticity and damage evolution. The damage evolution is a complicated phenomenon to account for and to characterize. It becomes even more complicated if the damage evolution at different environmental conditions is considered.

The objectives of this work is to study experimentally and theoretically the effect of extreme environmental conditions on damage evolution and lifetime of long fiber polymer based composites. The altered temperature and exposure to a salt water are the extreme environmental conditions that are considered in this study. The ambient room conditions are selected as a reference environmental conditions with respect to which the mechanical properties of the considered materials at extreme conditions will be analyzed and compared. At this point of the study, the material properties at the reference conditions have been studied experimentally and theoretically.

The continuum damage mechanics is a promising approach to develop thermodynamically consistent formulation of constitutive law for media with damage. This would account for effects of temperature changes (non-isothermal conditions) and environmental (moisture) conditions. This is because the additional internal state variables (ISV), that would account for considered phenomenon, can be included. The thermodynamically consistent damage dependent lamination theory, formulated by D.H. Allen [1,2], is used at this point of the study, to describe inelastic response to the applied load for a long fiber composite laminates at reference conditions (isothermal environmental conditions). The transverse cracking in UD composites is a rather well described damage mechanism in general. It is also discussed in details by authors of [1,2] within this approach. In this study, the internal state variable that accounts for damage of UD composite in fiber direction is formulated within the terms of the theory, and the function that describe the corresponding internal state variable is determined experimentally. The experimentally determined functions of ISV are used to predict the lifetime of laminates with arbitrary layups. The stiffness degradation is associated with corresponding damage evolution providing the tool for experimental determination of ISV accounting for considered damage.

Theory

Thermodynamic constraints on system with local damage

The constitutive relationships for media with cracks has been developed by [1,2,3,4] and herein used in form as

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} + I_{ijkl}^{\xi}\alpha_{kl}^{\xi}, \tag{1}$$

where I_{ijkl}^{ξ} is damage dependent moduli, and $\xi = 1,...,N$ accounts for different damage modes. The locally averaged internal state variables associated with energy dissipation due to the cracking is defined as first proposed by [5,6] and utilized by [1,2,3,4] as

$$a_{kl} = \frac{1}{V}\int_{S_2} u_k n_l dS_2, \tag{2}$$

where α_{kl} are components of internal state variable tensor, V is a local volume in which statistical homogeneity can be assumed, u_k and n_l are crack face displacement and normal respectively, S_2 is a crack surface area.

The reduction of the number of unknown constants in (1) can only be done by specifying the material symmetry and specific damage modes of interest. By utilizing the conditions of the symmetry of stress and strain tensors, leading to symmetry in $C_{ijkl} = C_{jikl}$, $I_{ijkl}^{\xi} = I_{jikl}^{\xi}$, and $C_{ijkl} = C_{ijlk}$ respectively, the number of unknown constants in (1) are reduced. Further, the symmetry conditions for compliance tensor, $C_{ijkl} = C_{klij}$ are provide by considering the second derivative of free energy for uncracked body. If the tensor of compliance is considered symmetric, $C_{ijkl} = C_{klij}$, this type of symmetry applies only to the compliance matrix. It is not required for damage tensor, I^{ξ}_{ijkl}. Material symmetry can be utilized for further simplification of constitutive equations, if the transverse isotropy is considered.

Further, the shape and size of damage tensor, I^{ξ}_{ijkl} have to be determined for each damage mode ξ in particular. It is assumed that damage from loading in fiber direction induces orthotropy in three principle planes. The same is assumed by D. Allen in case of transverse matrix cracking, namely transverse cracking also induces orthotropy in three principle planes. Therefore, the damage tensor I^{ξ}_{ijkl}, is an orthotropic tensor with 15 unknown constants given in material symmetry axis, where $\xi = 1$ accounts for induced damage due to loading in x_1 principal direction (fiber direction), and $\xi = 2$ will be used to account for transverse matrix cracking.

The general constitutive relationships (1) can be written using the Voigt notation [1,2] as

$$\sigma_i = C_{ij}\varepsilon_j + I^{\xi}_{ik}\alpha^{\xi}_{k} . \tag{3}$$

Equations for the laminate

If the constitutive relationships of single ply are given by (3), the constitutive relationships for general laminate with damage are obtained and written in the matrix form

$$\begin{aligned}\{N\} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \{\sigma\} dz \\ &= [A]\{\varepsilon\} + [B]\{k\} + \left\{f^{(D)}\right\} + \left\{f^{(1)}\right\} + \left\{f^{(2)}\right\} + \left\{f^{(3)}\right\},\end{aligned} \tag{4}$$

where extensional stiffness matrix is defined as

$$[A] = \sum_{k=1}^{n} [Q]_k \left(z_k - z_{k-1}\right),$$

and the coupling stiffness matrix of the laminate is given as

$$[B] = \frac{1}{2}\sum_{k=1}^{n} [Q]\left(z_k^2 - z_{k-1}^2\right).$$

The different forms of damage that are accounted for within the (4), are represented by $\left\{f^{(D)}\right\}$, $\left\{f^{(1)}\right\}$, $\left\{f^{(2)}\right\}$, and $\left\{f^{(3)}\right\}$, corresponding to delamination, fiber fracture, transverse cracking and shear debonding respectively. The fiber fracture that takes place due to the loading in fiber direction is only damage mode that will be addressed within this study. Therefore, the details regarding the $\left\{f^{(D)}\right\}$, $\left\{f^{(2)}\right\}$, and $\left\{f^{(3)}\right\}$ are left out of the scope of this paper. The effect of the fiber fracture that takes place due to the loading in fiber direction is described by

$$\left\{f^{(1)}\right\} = \sum_{k=1}^{n} \left[I^{(1)}\right]_k \left(z_k - z_{k-1}\right)\left\{\alpha^{(1)}\right\}_k ,$$

where $\left\{f^{(1)}\right\}_k$ is a damage stiffness tensor of k^{th} layer, and $\left\{\alpha^{(1)}\right\}_k$ is internal state variable accounting for strain perturbations due to the considered damage. Generally, $\left\{\alpha^{(1)}\right\}_k$ can be determined experimentally, or calculated using micromechanics approach. The specially designed, UD laminate in particular, must be used in order to determine $\left\{\alpha^{(1)}\right\}_k$ experimentally.

ISV for damage due to the loading in fiber direction, $\left\{\alpha^{(1)}\right\}_k$

The accurate predictions of damage evolution in fiber direction play crucial role in determination of fatigue lifetime of the laminated composite. As the applied stress is increased, additionally to the fatigue damage, the damage modes (mechanisms) associated with static loading must be considered, as well as possible coupling between both. The expected damage due static loading in fiber direction is schematically illustrated in Figure 1.

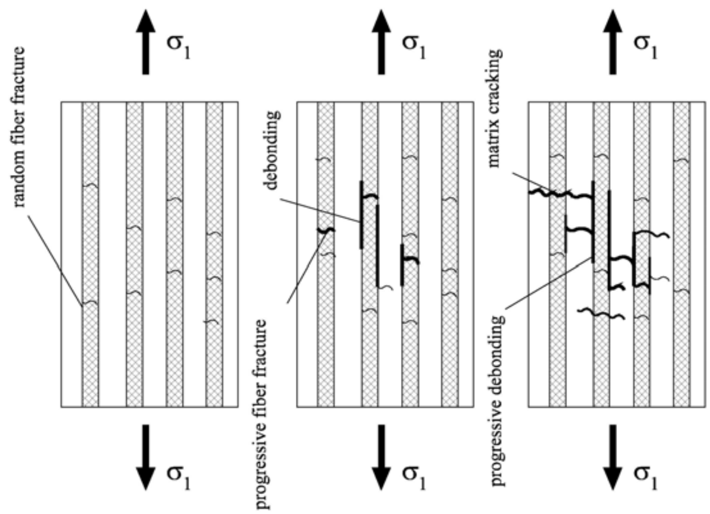

Figure 1: Damage development in composite laminate loaded in fiber direction.

The ISV tensor may be considered as a pseudo-strain due to the cracking. For the considered damage, the only nonzero components of the tensor are

$$\left\{\alpha^{(1)}\right\} = \left\{\alpha_{11}^{(1)} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \alpha_{31}^{(1)} \quad 0 \quad \alpha_{21}^{(1)}\right\}^T . \tag{5}$$

It means, that the considered damage will affect the strain in fiber direction as well as in-plane and out-of-plane shear strain. Meaning, $\alpha_{11}^{(1)}$ allows crack opening in mode-I, $\alpha_{31}^{(1)}$ and $\alpha_{21}^{(1)}$ allow in and out of plane movement (slip) of the crack surface against each other in mode-II. The crack normal has one component in direction 1, and crack orientation does not change during the loading.

Considering the constitutive equations for the layer (3) and applying the orthotropy symmetry constraints for C_{ij} and damage damage tensor I_{ik}^{ξ}, together with vector of internal state variable (5), the expression (3) can be simplified if the plane stress conditions (and neglecting of out of plane strain components) are assumed,

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0\\ C_{21} & C_{22} & 0\\ 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_6\end{Bmatrix} + \begin{bmatrix} I_{11}^{(1)} & 0 & 0\\ I_{21}^{(1)} & 0 & 0\\ 0 & 0 & I_{69}^{(1)}\end{bmatrix}\begin{Bmatrix}\alpha_{11}^{(1)}\\ 0\\ \alpha_{21}^{(1)}\end{Bmatrix}. \qquad (6)$$

It is easy to se that the $\alpha_{11}^{(1)}$ can be found experimentally as an only non-zero component of $\{\alpha^{(1)}\}$ if the specific, $\{\sigma\} = \{\sigma_1 \quad 0 \quad 0\}^T$ are applied.

Equations for unidirectional laminate (UD)

Let's consider the unidirectional laminate with fiber orientation in fiber direction, and formulate the constitutive relationships for this particular case. In case of static loading, $\{N\} = \{N_x \quad 0 \quad 0\}^T$, there is only one type of damage expected, and int can be described with the internal state variable $\alpha^{(1)}$, see section [*ISV for damage due to the loading in fiber direction*]. Further, the constitutive relationships for the considered laminate can be written in a compact matrix form as

$$\{N\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \{\sigma\}dz = [A]\{\varepsilon\} + \{f^{(1)}\}. \qquad (7)$$

For UD laminate $[0_n]$, $[I^{(1)}]_k = [I^{(1)}]\forall k$, $[\alpha^{(1)}]_k = [\alpha^{(1)}]\forall k$ and t_{k}=t\forall k . It simplifys (7) to form

$$\{N\} = [A]\{\varepsilon\} + [I^{(1)}]\{\alpha^{(1)}\}h\,. \qquad (8)$$

Considering loading conditions in fiber direction, $\{N\} = \{N_1 \quad 0 \quad 0\}^T$, (8) is written as

$$N_1 = A_{11}\varepsilon_1 + A_{12}\varepsilon_2 + I_{11}^{(1)}\alpha_{11}^{(1)}h \qquad (9)$$

$$0 = A_{21}\varepsilon_1 + A_{22}\varepsilon_2 + I_{21}^{(1)}\alpha_{11}^{(1)}h \qquad (10)$$

We have now two equations with three unknowns in them. The number unknowns could be reduced by using, $-C_{ijkl} = I_{ijkl}$ [1,2], and $A_{ij} = C_{ij}h$. Than (9,10) becomes as

$$N_1 = A_{11}\varepsilon_1 + A_{12}\varepsilon_2 + C_{11}\alpha_{11}^{(1)}h\,, \qquad (11)$$

where A_{ij}, and C_{ij} are elastic constants, that can be calculated using micromechanics or can be determined experimentally. $\alpha_{11}^{(1)}$ is internal state variable that accounts for the damage that occur from static loading in fiber direction.

Determination of $\alpha_{11}^{(1)}$

The stiffness degradation as a function of applied strain can be utilized in order to determine $\alpha_{11}^{(1)}$ experimentally. Introducing the definition of the stiffness

$$E_1(\varepsilon) \equiv \frac{1}{h}\frac{\partial E_1}{\partial \varepsilon_1}, \tag{12}$$

and using (12), (11) results into

$$\begin{aligned} E_1(\varepsilon_1) &= \frac{1}{h}A_{11} + \frac{1}{h}A_{12}\frac{\partial \varepsilon_2}{\partial \varepsilon_1} - \frac{1}{h}A_{11}\frac{\partial \alpha_{11}^{(1)}}{\partial \varepsilon_1} \\ &= \frac{1}{h}A_{11} + \frac{1}{h}\frac{\partial \alpha_{11}^{(1)}}{\partial \varepsilon_1}\left(\frac{A_{12}A_{21}}{A_{22}} - A_{11}\right) - \frac{1}{h}\frac{A_{12}A_{21}}{A_{22}}, \end{aligned} \tag{13}$$

from where

$$\alpha_{11}^{(1)}(\varepsilon_1) = \frac{h\int_{\varepsilon_1} E_1(\varepsilon_1)d\varepsilon_1 + \frac{A_{12}A_{21}}{A_{22}}\varepsilon_1 - A_{11}\varepsilon_1}{\left(\frac{A_{12}A_{21}}{A_{22}} - A_{11}\right)}. \tag{14}$$

Results and discussions

The stiffness degradation of UD laminate, $[0_4]_T$, is measured experimentally using quasi-static tensile loading-unloading test. The whole hysteresis loop of the i^{th} loading-unloading cycle is used to calculate the Young's modulus, $E_1(\varepsilon_1)$ for corresponding applied strain. It was observed, that the stiffness degradation measured is consistent for all individual specimens.

The measured stiffness degradation is normalyzed, $E_1(\varepsilon_1)/E_1(\varepsilon_1 = 0)$ and described by a second order polynomial, see Figure 2.

In the tensile test, there are limits of how close the maximum damage state can be characterized. In this work, the damage state is characterized up to applied strain $\varepsilon_1 = 2.25[\%]$.

The rapid fracture process takes place within very short interval of applied strain after this point. The maximum strain to failure was measured $\varepsilon_1 = 2.53[\%]$. The fracture that leads to the final failure is localized and would not represent the overall material property.

Further the obtained function for $E_1(\varepsilon_1)/E_1(\varepsilon_1 = 0)$ can be used into (14) in order to calculate $\alpha_{11}^{(1)}(\varepsilon_1)$. The calculated values of $\alpha_{11}^{(1)}(\varepsilon_1)$ are given in Figure 3.

In order to validate the outlined approach and measured $\alpha_{11}^{(1)}(\varepsilon_1)$, the strain-stress curve of independent tensile test of UD laminates, $[0_4]_T$, can be predicted by using the obtained function of $\alpha_{11}^{(1)}(\varepsilon_1)$ into the constitutive equations for laminate, (4). The results of predictions are given in Figure 4.

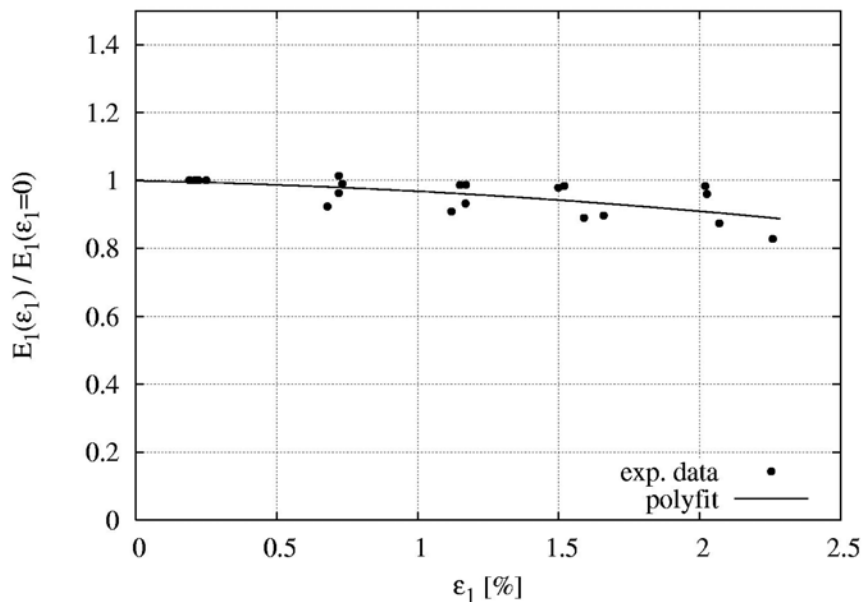

Figure 2: Stiffness degradation of UD composite. The second order polynomial, $y = a_2 x^2 + a_1 x + a_0$ *is used for approximation. The calculated constants are,* $a_0 = 1.0$ *,* $a_2 = -1.76$ *and* $a_2 = -136.9$.

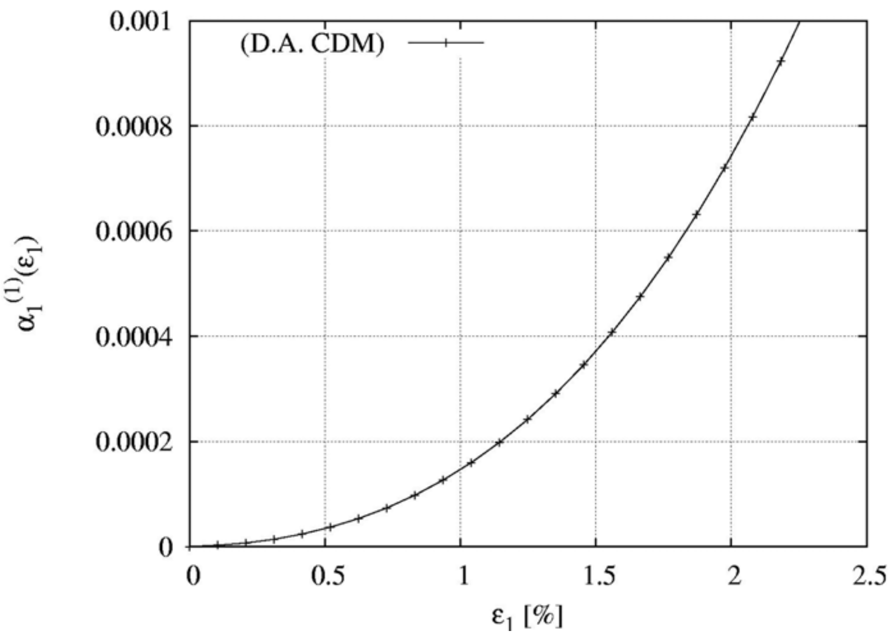

Figure 3: Calculated ISV $\alpha_{11}^{(1)}(\varepsilon_1)$.

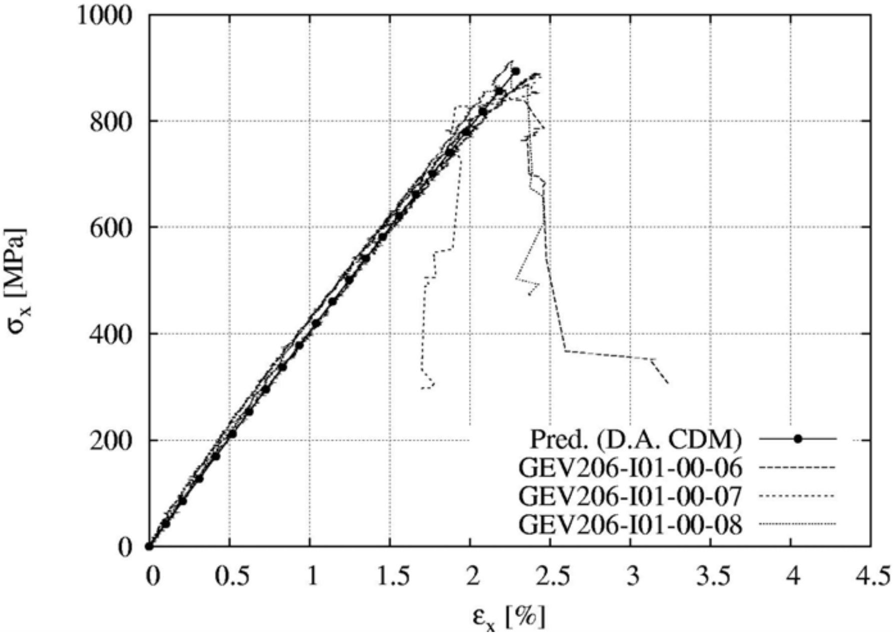

Figure 4: Predictions of strain-stress behavior of UD laminates in tensile test compared with experimental data.

Considering the limits of the damage evolution characterization, there is a considerable good agreement between the predicted strain and stress values and experimental data. Since the loading conditions, $\{N\} = \{N_1 \quad 0 \quad 0\}^T$, are applied for the test, the inverse constitutive equations must be used in order to predict strain-stress curve. The inverse constitutive relationships in general are given in the Appendix.

Concluding remarks

The continuum damage mechanics approach is used to account for and characterize the damage evolution in long fiber laminated composites. The internal state variable evolution law associated with damage in fiber direction is formulated in terms of used theory, and determined experimentally. The experimentally determined evolution law of internal state variable is used in damage dependent constitutive relationships for laminate in order to predict the static strain - stress behavior of arbitrary laminates.

The measurements of the stiffness degradation are used to determine the corresponding internal state variable function. The relationships between particular ISV and corresponding stiffness degradation is formulated for considered damage mechanism.

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Appendix

$$\{\varepsilon\} = [A']\{N\} + [B']\{M\} + \{f^{(D)}\}' + \{f^{(1)}\}' + \{f^{(2)}\}', \qquad (15)$$

where

$$[A'] = [A^*] - [B^*][D^*]^{-1}[C^*]$$
$$[B'] = [B^*][D^*]^{-1}$$
$$\{f^{(D)}\}' = -[B^*][D^*]^{-1}\{g^{(D)}\}^* + \{f^{(D)}\}^*$$
$$\{f^{(1)}\}' = -[B^*][D^*]^{-1}\{g^{(1)}\}^* + \{f^{(1)}\}^*$$
$$\{f^{(2)}\}' = -[B^*][D^*]^{-1}\{g^{(2)}\}^* + \{f^{(2)}\}^*$$

and

$$[A^*] = [A]^{-1}$$
$$[B^*] = [A]^{-1}[B]$$
$$\{f^{(D)}\}^* = -[A]^{-1}\{f^{(D)}\}$$
$$\{f^{(1)}\}^* = -[A]^{-1}\{f^{(1)}\}$$
$$\{f^{(2)}\}^* = -[A]^{-1}\{f^{(2)}\}$$

There are a shorter alternative way to obtain the inverse relationships, nevertheless, the general form given above is used herein.

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