The Multislope Model

A new description for the fatigue strength of glass fibre reinforced plastic

G.K. Boerstra

Date: 10-02-2006

Document nr: D0003303

1 Introduction

For the life time calculations of structures of glass fibre reinforced plastics such as rotor blades of wind turbines it is essential to have access to accurate information about the fatigue resistance of the material. These data should comprise the number of cycles to failure under a specific combination of stress amplitude and mean stress. The fatigue data can be presented as a descriptive model which is essentially a three dimensional figure of which the familiar diagrams such as the S-N lines and the Constant-Life lines (or shortly CL lines) are two-dimensional sections. The S-N lines are sections parallel to the N axis and the CL lines are presented in the plane perpendicular to this axis.
Traditionally these figures are kept as simple as possible. S-N lines are preferably straight on log-log scale while the CL lines are triangular (Goodman diagrams). The advantages were obvious in pre-computer times but nowadays more sophisticated models with more parameters may be preferred if this can improve the accuracy of the fatigue life prediction.
This report introduces a new method for combining the measurements together with a new type of three-dimensional model and compares it with the traditional model in terms of accuracy that can be achieved.

2 Data processing in the traditional way

Test pieces loaded with various stress levels will be grouped together with the same stress ratio R. This can be e.g. one group with R=-1, one with R=10 and one with R=0.1. Per group the results are plotted in a graph with the achieved number of cycles on the horizontal axis and the stress amplitude on the vertical axis. Through the data points a regression line is fitted that is preferably linear when both axes are scaled logarithmically. This result in three different regression lines; each with its own slope. The stress values of the regression lines at round numbers of cycles are read from these graphs and plotted in CL lines with the mean stress on the horizontal axis and the amplitude on the vertical axis. For each number of cycles the stress points are connected together with the static values on the horizontal axis. The result looks rather odd (see fig. 1) and seems not suited for any mathematical description.

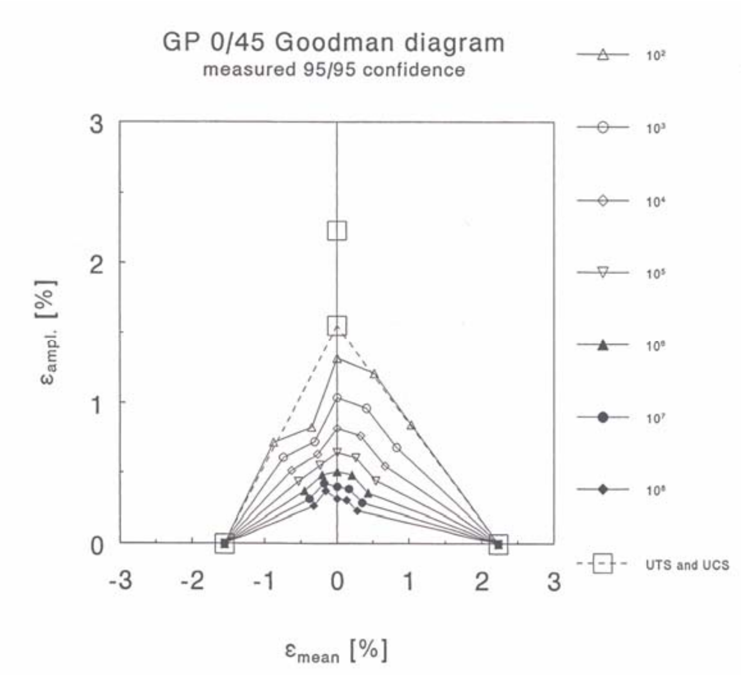

Fig1. (Source ref. [4]).

The only sensible way to fit the CL lines into a mathematic model is to draw triangles with an apex that decreases with the same power law as the S-N line for R=-1 (see fig 2).

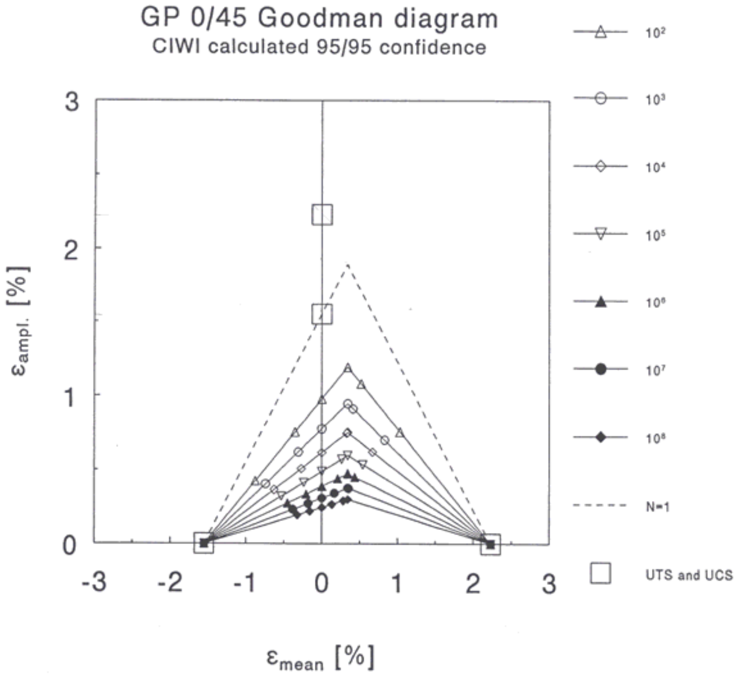

Fig2. (Source ref. [4]).

It is easy to see that the original figure has undergone a drastic change in order to find something that can be used for mathematical operations. The result is however far from satisfying. For this reason the model has to be expanded with more parameters.

3 The Multislope model

CL lines are not necessarily triangular (except for the CL line for N=1). For ductile materials, such as steel, Gerber already proposed a parabola and the ASME even suggested using an ellipse. The latter is quite unrealistic since at high mean stresses the ellipse always exceeds the limit of the N=1 triangle.
Taking the Gerber shape as a starting point and replacing the exponent 2 by a variable exponent offers more possibilities. It seems most appropriate to take different exponents for the tension and the compression side: αt for the tension and αc for the compression side. This yields the general formulas:

For $S_m>0$: $S_{ap} = S_{Ap} * (1 – (S_m/UTS)^{\alpha T})$.....{1}

For $S_m<0$: $S_{ap} = S_{Ap} * (1 – (S_m/UCS)^{\alpha C})$.....{2}.

The second expansion of the model will be made by introducing variable slopes for the S-N lines with various mean stress values. As can be seen from fig 1 at the tension side of the diagram the CL lines are spread more widely, suggesting a steep S-N line with small m value and on the compression side the CL lines are more concentrated with a flatter S-N line and higher value for m.
Practical solutions for the dependency of m can be applying a linear or an exponential relation with the mean stress S_m:

$m = m0 * (1 - S_m/D)$ .....{3} or $m = m0 * e^{(-Sm/D)}$ .....{4}.

It can be observed that for the triangular model the slope of the S-N lines for different R-values is already variable. The slope is minimal for R=-1 and rises on either side to ∞ when R approaches +1.

When both expansions are combined it becomes necessary to choose a reference number of cycles for which the CL line will be defined by the parameters αT and αC. For all other numbers the shapes of the CL lines follow from the defined change in S-N slope. How critical this choice of the referential number is will be analysed in chapter 4.

When constructing the model for a specific material first the static strengths UTS and UCS have to be determined. Than fatigue tests on coupons with various values of stress amplitude S_a and mean stress S_m will be performed. Now a preliminary model can be based on a chosen value for N_p, an arbitrary choice between the linear or the exponential relation between m and S_m and estimates for the values for the parameters m0, D, αT and αC.

For the derivation of the best fitting diagram using all possible parameters the method of the least squares is followed both for the stress amplitude and for the number of cycles.
The rationale for using both is as follows:

On the tensile side of the mean values the S-N slope is rather steep and on the compressive side more gentle. This will cause that measuring points on the compressive side will become more dominant than those on the tensile side when the least squares of the number of cycles is used for determination of the best fitting regression line. Likewise can be seen that points on the tensile side will become more dominant than those on the compressive side when the least squares of the stress is used for the regression analysis. To balance the influence of both directions with each other the regression can best be performed by minimising the combination of both deviations.

First the deviation on the Stress amplitude ΔSa is calculated:

- Calculate for each S_m the expected value for m with {3} or {4} and project each measured point along the S-N slope onto the N_p plane with $S_{ap} = S_a * (N/N_p)^{(1/m)}$
- Calculate for each point projected in the N_p plane de difference between the logarithm of the projected amplitude of the preceding step and the logarithm of the amplitude value following from the model for this S_m, using {1} or{2}.
  $\Delta Sa = \ln(S_{ap}) - \ln(S_{ap,\, mod})$

Secondly the deviation on the number of cycles Δn is calculated:
Calculate for each measured point Δn as the difference between the realised number of cycles and the predicted number of cycles N_e
$\Delta n = \ln(N) - \ln(N_e)$

Finally combine ΔSa and Δn to Δt such that within the S-N plane through each point the shortest distance to the S-N line is obtained:

$$\mathbf{\Delta t} = \text{sign}(\Delta Sa) * \sqrt{(1/(1/\mathbf{\Delta Sa}^2 + 1/\mathbf{\Delta n}^2))}.$$

The standard deviation of all values of Δt will now be called the total standard deviation SDt.
The best fit will be reached by minimising the SDt by varying the estimated parameters m0; D; αT and αC using the solver tool of a spreadsheet.

The value for S_{Ap} (the apex of the CL line in Np) will be calculated as follows:
Transpose each point S_{ap} in the N_p plane (see above) along the CL lines onto the Sm=0 axis with:

{1'} : $SAp = Sap / (1 - (Sm/UTS)^{\alpha T})$ or {2'} : $SAp = Sap / (1 - (Sm/UCS)^{\alpha C})$.

The average of all individual points will be used as the SAp in the model.

4 Case study with existing data

To test the efficiency of the expanded model a set of existing data is taken of fatigue tests on glass-polyester coupons measured at ECN and TU Delft (See Annex 1). These data are taken from the database FACT and already analysed in [2], [3] and [4]. It comprised material with 50% UD and 50% ± 45 fibres denoted as GP 0/45.
The static strength of this material was:
UTS = 370 MPa and UCS = 286 MPa when straining slow (0.02 mm/s) and for fast measuring (2 mm/s) these values were UTS = 445 MPa and UCS = 305 MPa. For fitting into the model the “slow” values were used.
The S-N line for R=-1 showed a slope of 1:10.

At first an investigation will be made which variety of the triangular model will be more appropriate: one with the apex at m=0 and the other with the apex symmetrically between UTS and UCS (The GL shape of fig.2).
When the measured points were projected along a 1:10 slope onto the N=1 plane the result looks as follows:

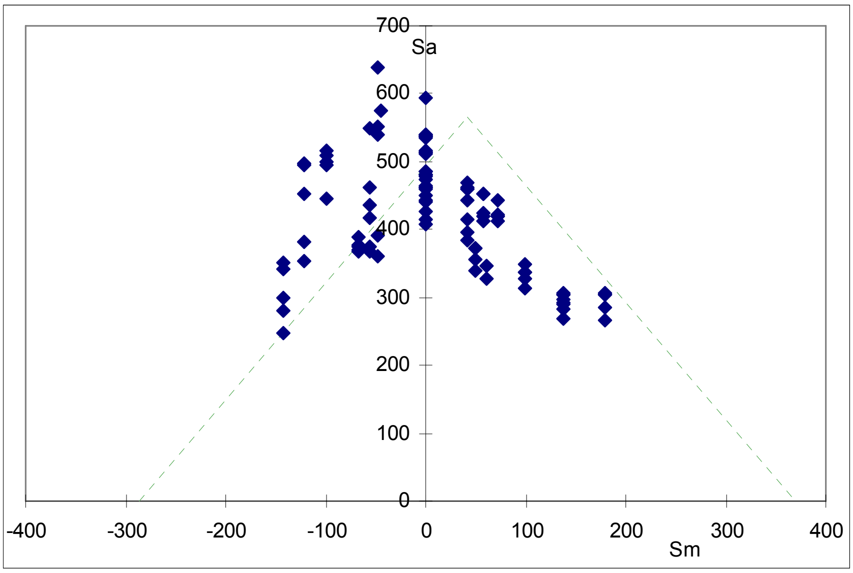

Fig. 3

It can be seen easily that the triangle with the symmetric apex gives a poor fit. This is also quantified by the values for SDt for both solutions.
Optimising gave the following values:

	Apex at Sm=0	Symmetric apex
m0	10.09	10.62
SDt	0.172	0.253
S_{A1}	389	473.4

Table 1

Taking the model with the S_m=0 apex as a starting point the influence of adding various parameters of the new model is investigated again with optimising to a lowest value for SDt. The most sophisticated expanded model with all parameters free is compared with the more simple models with restricted parameters with αT and αC equal to 1 or only one slope. For the variable slope versions the exponential relation between S_m and m was used according formula {4}.
The result can be summarised as follows:

	Constant slope, Curved CL lines	Variable slope, Straight CL lines for N_p	All parameters free
N_p	1	2245	100
m0	10.08	10.47	10.54
αT	1.09	1.00	2.06
αC	2.90	1.00	1.04
D	n.a.	234	244
SDt	0.126	0.0945	0.0787
S_{A1}	389	358	389

Table 2

As can be seen from table 2, giving up the triangular shapes already gives a remarkable improvement in the scatter just as introducing a variable slope. Combination of both expansions gives the best results, as was expected.
The choice of N_p, the reference number for the projection plane, is not very critical for the achieved SDt values. For all values between 1 and 10^4 the SDt was about 0.08 with almost the same values for m0, and D. Only the values for αT and αC adapted themselves to the chosen number N_p:

N_p	1	10	100	500	1000	10000
m0	10.47	10.51	10.54	10.56	10.57	10.62
αT	20.00	3.37	2.06	1.52	1.36	1.03
αC	0.89	0.94	1.04	1.16	1.22	1.46
D	314	268	244	245	250	278
SDt	0.0824	0.0798	0.0787	0.0792	0.0797	0.0821

Table 3

From the point of view of the lowest standard deviation it is best to take N_p equal to 100. Furthermore with this value the bundle of CL lines showed the best resemblance with figure 1:

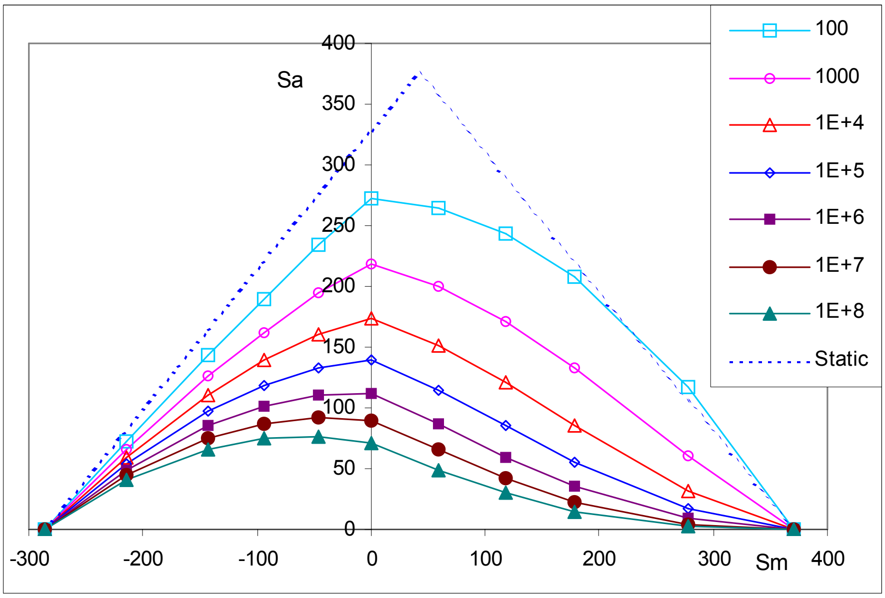

Fig 4

Remark: The amplitude value of the static limit is based on the “fast” static values.

The effectiveness of the model can be illustrated when all measuring points are projected along the straight SN slopes onto the N_p plane together with the CL line that follows from the model:

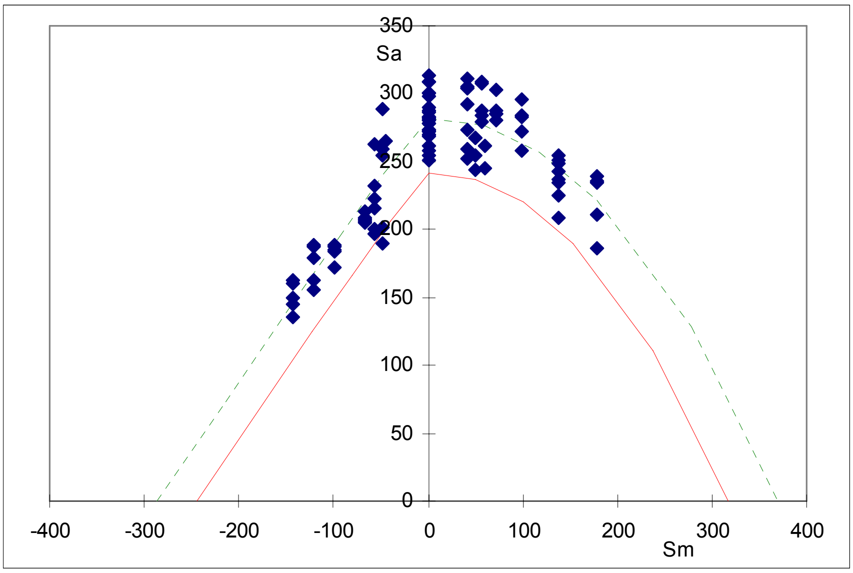

Fig. 5.

Below the measuring points the 95/95 limit, calculated from the Standard Deviation on the stress amplitude as per the formula of GL (see [1]), is drawn as a reference.

A second way for illustration of the effectiveness is the transposition of all measuring points along the CL lines to the S_m=0 plane and to plot them together with the calculated S-N line for R=-1:

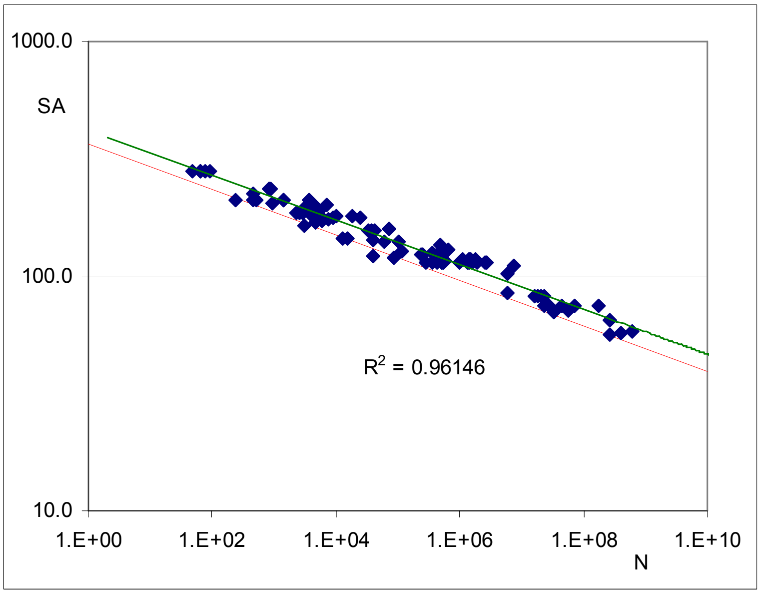

Fig. 6

Again the 95/95 limit is drawn as a reference.
From the figures 5 and 6 can be seen that the fit of the model is remarkably well. Even the test results with failure cycles below 100 can still be described accurately in spite of the fact that the model starts violating the static limits for higher mean stresses (see fig 4).

The accuracy of the formula for m can best be demonstrated by calculating for each measured point the slope of the log-log line in the constant S_m plane which connects the point with the CL line for N_p .

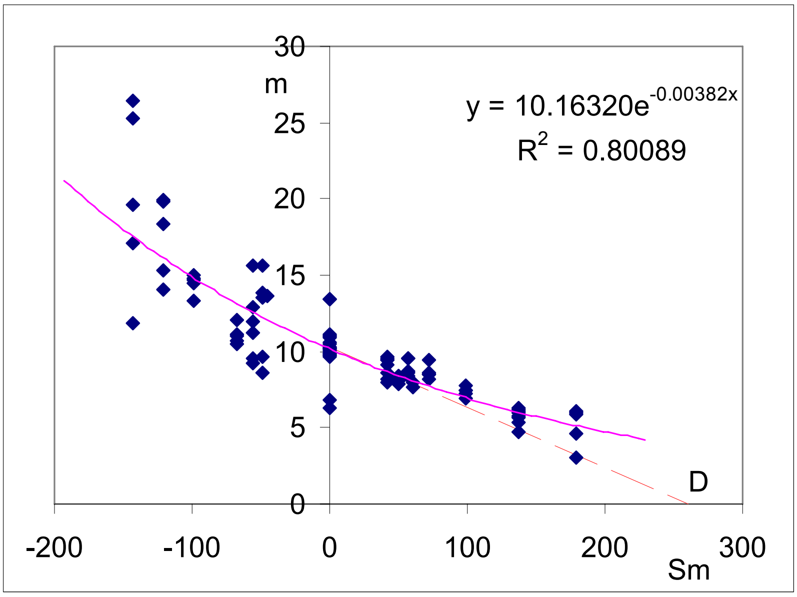

Fig 7.

The regression through the data points returns the values m0=10.16 and D= 1/0.00382 = 261 MPa. The graph clearly shows the meaning of the parameter D as the starting point on the S_m-axis of the tangent to the curve at m0.

.

5 Conclusions and Discussion

The proposed Multislope model gives a very good fit with the measured data points. The standard deviation of all points is more than half the standard deviation in the traditional model with triangular CL lines and a constant S-N slope.
Expressed in terms of the 95/95 value this results in an admissible stress, which is 25% higher than when using the simple triangular model.

The model is relatively insensitive for “good” or “bad” static values. The αT and αC can simply flex the CL curves to the best fit whatever the static values are. This certainly is an advantage considering the difficulty to perform reliable compressive tests.

The model uses 7 parameters for its description. One can argue that this lack of simplicity or elegance is proof that it can never be a true description of reality.
The model however does not pretend to give a theory explaining why failure takes place but is merely a practical description for engineering purposes.

A drawback of the model could be that it looses accuracy for very low numbers of cycles and ignores the static strength. This could be solved by introducing S-N lines shaped as Sendeckyj lines but immediately the question will rise what static strength is to be used; the “slow” or the “fast” value?
On the other hand one may also state that fatigue has nothing to do with static strength and it is allowed to split the problem in two. The fatigue model can be declared valid for numbers of cycles larger than 100

and will be based on measurements with this minimum lifetime. A static strength verification must be performed separately using the static values.

The model fitting is based on reducing the scatter in stress as well as in the number of cycles. Normally for the fit of the S-N line for one R-value a regression calculation of N on S is used since S is the independent variable and N is the dependent.
This is however not completely true since only the average stress is known. Internal stress distribution can be different for various test coupons so local stresses can cause scatter as well. Another rationale is that also the calculation of the 95/95 limit is performed on stress level using the SD on the stress.

6 References

[1] Germanischer Lloyd, *Regulations for the Certification of Wind Energy Conversion Systems,* Hamburg, 2003.

[2] Bach, P.W. , *Fatigue lifetime of glass-polyester laminates for wind turbine blades,* ECN-C-94-020, March 1994.

[3] Bach P.W. en De Smet B.J.*, Database FACT, Fatigue of composites for wind turbines*, ECN-C-94-045, August 1994.

[4] Bach P.W. en De Smet B.J., *Life time predictions of glass fibre reinforced polyester with database FACT* , ECN-C-94-044, August 1994.

7 Nomenclature

D	Skewness parameter for the dependency of m.
m0	Slope of S-N line on log-log scale for $S_m=0$
m	Slope of S-N line for mean stress S_m
N	Number of cycles to failure
N_e	Expected number of cycles based on model
N_p	Reference number of cycles for defined shape of CL line
R	Stress ratio: S_{min} / S_{max}.
S_a	Stress amplitude of load cycle
S_{A1}	Apex Amplitude for N=1 and $S_m=0$
S_{Ap}	Apex Amplitude for N_p cycles and $S_m=0$
S_{ap}	Amplitude for N_p cycles and mean stress S_m
S_m	Mean stress of load cycle
SDt	Standard deviation of Δt
αC	Exponent for CL line for $S_m<0$
αT	Exponent for CL line for $S_m>0$
ΔSa	Logarithmic deviation of stress amplitudes S_{ap}.
Δn	Logarithmic deviation of number of cycles.
Δt	"Shortest distance" combination of ΔSa and Δn.

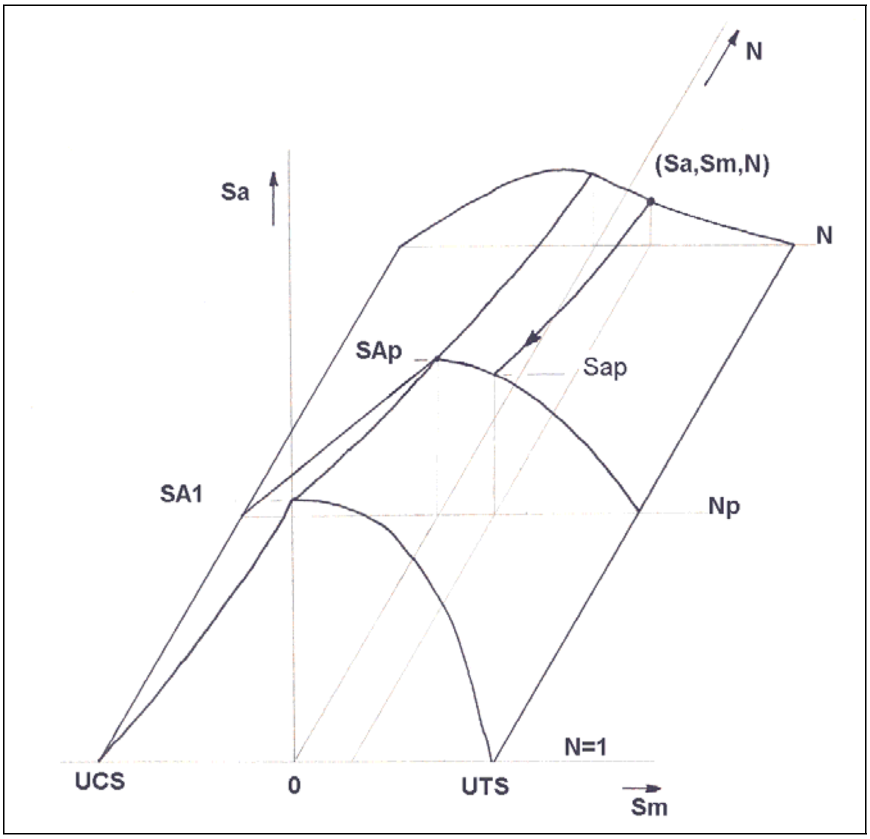

Fig.7.

Annex 1 Fatigue data FACT database.

Code	Sm	Sa	R	N	Code	Sm	Sa	R	N
ecn895	0	75	-1	23532924	ecn300	-56.25	131.25	-2.5	105012
ecn896	0	115	-1	430305	ecn352	-56.25	131.25	-2.5	498212
ecn897	0	115	-1	501860	ecn408	-48.75	113.75	-2.5	39556
ecn898	0	115	-1	503105	ecn348	-48.75	113.75	-2.5	84506
ecn899	0	115	-1	536305	ecn345	-48.75	113.75	-2.5	1566656
ecn900	0	115	-1	972255	ecn068	-48.75	113.75	-2.5	1930642
ecn901	0	115	-1	1385500	ecn347	-48.75	113.75	-2.5	7368156
ecn902	0	210	-1	450	ecn344	-45	105	-0.4	5930012
ecn903	0	210	-1	517	ecn331	42	98	-0.4	281506
ecn904	0	210	-1	1395	ecn290	42	98	-0.4	355056
ecn905	0	210	-1	1403	ecn291	42	98	-0.4	570556
ecn906	0	210	-1	1421	ecn332	42	98	-0.4	1030406
ecn907	0	210	-1	3701	ecn289	42	98	-0.4	1422262
ecn908	0	210	-1	3715	ecn288	42	98	-0.4	1448856
ecn909	0	280	-1	48	ecn287	42	98	-0.4	1493756
ecn910	0	280	-1	64	ecn292	42	98	-0.4	1748406
ecn911	0	280	-1	66	ecn333	57	133	-0.4	32856
ecn912	0	280	-1	78	ecn294	57	133	-0.4	37206
ecn913	0	280	-1	92	ecn336	57	133	-0.4	41856
tud11	0	115	-1	1300000	ecn335	57	133	-0.4	73706
tud12	0	115	-1	1400000	ecn334	57	133	-0.4	74256
tud13	0	65	-1	2.67E+08	ecn297	72	168	-0.4	3800
tud16	0	65	-1	2.64E+08	ecn299	72	168	-0.4	4446
tud04	0	75	-1	1.74E+08	ecn296	72	168	-0.4	4457
tud09	0	75	-1	44000000	ecn295	72	168	-0.4	4693
tud18	0	75	-1	28000000	ecn298	72	168	-0.4	7112
tud19	0	75	-1	71000000	tud05	50	41	0.1	2.69E+08
ecn285	-143	117	10	956	tud06	50	41	0.1	6.03E+08
ecn329	-143	117	10	3102	tud07	50	41	0.1	3.97E+08
ecn293	-143	117	10	5397	tud08	50	41	0.1	6.03E+08
ecn328	-143	117	10	18731	ecn919	60.5	49.5	0.1	33593418
ecn284	-143	117	10	24450	ecn920	60.5	49.5	0.1	57730636
ecn066	-121	99	10	117056	ecn921	99	81	0.1	251697
ecn286	-121	99	10	231656	ecn922	99	81	0.1	366524
ecn283	-121	99	10	1113306	ecn923	99	81	0.1	479884
ecn067	-121	99	10	2512453	ecn924	99	81	0.1	487314
ecn327	-121	99	10	2678706	ecn925	99	81	0.1	652413
ecn315	-99	81	10	6027280	ecn926	137.5	112.5	0.1	3006
ecn065	-99	81	10	16158548	ecn927	137.5	112.5	0.1	4806
ecn326	-99	81	10	17879606	ecn928	137.5	112.5	0.1	6056
ecn281	-99	81	10	20733224	ecn929	137.5	112.5	0.1	6462
ecn330	-99	81	10	23813962	ecn930	137.5	112.5	0.1	7555
ecn340	-67.5	157.5	-2.5	2347	ecn931	137.5	112.5	0.1	8855
ecn341	-67.5	157.5	-2.5	2557	ecn932	137.5	112.5	0.1	9255
ecn339	-67.5	157.5	-2.5	2851	ecn933	137.5	112.5	0.1	10005
ecn342	-67.5	157.5	-2.5	2927	ecn934	178.75	146.25	0.1	245
ecn343	-67.5	157.5	-2.5	3973	ecn935	178.75	146.25	0.1	471
ecn349	-56.25	131.25	-2.5	12806	ecn936	178.75	146.25	0.1	820
ecn338	-56.25	131.25	-2.5	15356	ecn937	178.75	146.25	0.1	844
ecn337	-56.25	131.25	-2.5	39456	ecn938	178.75	146.25	0.1	898
ecn351	-56.25	131.25	-2.5	61356				0.1	